## Open Problems

Workshop on Graph Drawing and Graph Algorithms 2013
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## Coin-graph Recognition

Q. What are the graphs that come up by touching coins?
Q. Can we recognize coin graphs in polynomial time?
Q. Is there any nontrivial sufficient condition on a planar graph to be a coin graph?


A graph of $n$ vertices is a touching unit circle graph or coin graph if it can be produced by $n$ non-overlapping circles in contact, where each circle represents a node and each pairwise contact represents an edge.

## Known Result:

Every planar graph can be represented as a contact graph of circles (Koebe's Theorem).

## Polyline Grid Drawing

Q. Does every outerplanar graph admit a polyline grid drawing in $\mathrm{O}(n \log n)$ area with at most two bends per edge?


Outerplanar graph


Polyline grid drawing

Known Result:
Every outerplanar graph admits a polyline grid drawing in $\mathrm{O}(n \log n)$ area with at most three bends per edge.

## Minimum Segment Drawing

Q. Is the problem solvable in polynomial time if the input graphs are plane 3 -trees, even when the maximum degree is bounded by a fixed constant?


Minimum segment drawing


Plane 3-tree

Known Results: -NP-hard in general.

- Polynomial time solvable for series parallel graphs with maximum degree 3 and 3-connected cubic graphs.


## Point-set Embedding

Q. Given a tree of $n$ vertices and a set of $n$ points in general position, is it possible to decide in polynomial time whether the tree admits a point set embedding such that all the leaves can be joined in order with straight line segments to form a cycle?


> Point set embedding of $T$ on $P$

Tree $T$
Point set $P$

## Consequence:

Polynomial time decision algorithm for point-set embedding of Halin Graphs.

## Straight-line Grid Drawing

Q. Characterize the planar graphs that admit a straight-line grid drawing $\Gamma$ s.t for every pair of vertices $(u, \downarrow)$ in $G$, a shortest path between $u$ and $v$ in $G$ is also a shortest path in $\Gamma$.


Straight-line grid drawing


Unit edge length graph

Known Result:
Partial results come from unit edge length graph drawing.

## Graph Representation

## (Touching Triangle Representation)

Q. Given a planar graph, is it possible to decide whether it admits a straight-line drawing in polynomial time s.t all facial polygons are drawn as triangles?


A straight-line drawing of a planar graph, with all facial polygons drawn as triangles

## Known Result:

Necessary and sufficient conditions for 3-connected plane graphs (but no polynomial-time algorithm is known to verify these conditions.)

