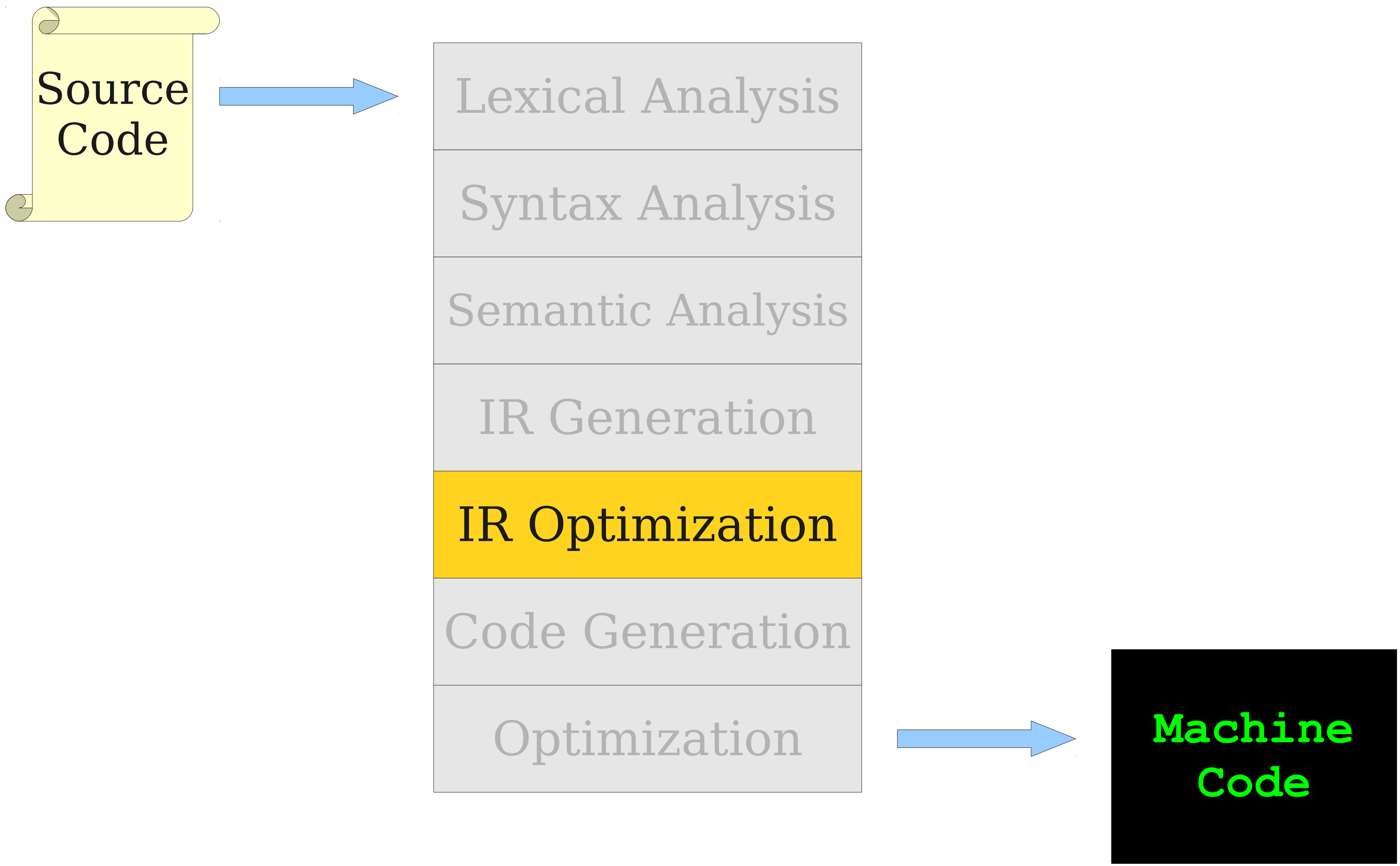
Global Optimization

Where We Are


Source Code
Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization
Machine Code

Review of Local Optimization

Review from Last Time

- A **basic block** is a series of IR instructions where
  - there is one entry point into the basic block, and
  - there is one exit point out of the basic block.
- Intuitively, a block of IR instructions that all must execute as a unit.
- A **control-flow graph** (CFG) is a graph of the basic blocks of a function.
- Each edge in a CFG corresponds to a possible flow of control through the program.

Review from Last Time

- A **local optimization** is an optimization of IR instructions within a single basic block.
- We saw five examples of this:
  - **Common subexpression elimination**.
  - **Copy propagation**.
  - **Dead code elimination**.
  - **Arithmetic simplification**.
  - **Constant folding**.

Review from Last Time

- Last time, we defined two analyses used in our optimizations.
- **Available expressions**: Track what variables are assigned which expressions.
  - Compute by walking forward across the values in a basic block.
- **Live variables**: Track what variables will eventually be used.
  - Compute by walking backward across the values in a basic block.

Available Expressions

```
a = b;
c = b;
d = a + b;
e = a + b;
d = b;
f = a + b;
```

Available Expressions

```
{ }
a = b;

c = b;

d = a + b;

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Available Expressions

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Another View of Local Analyses

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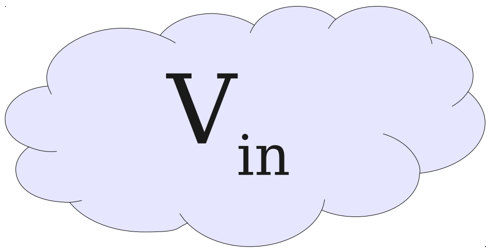

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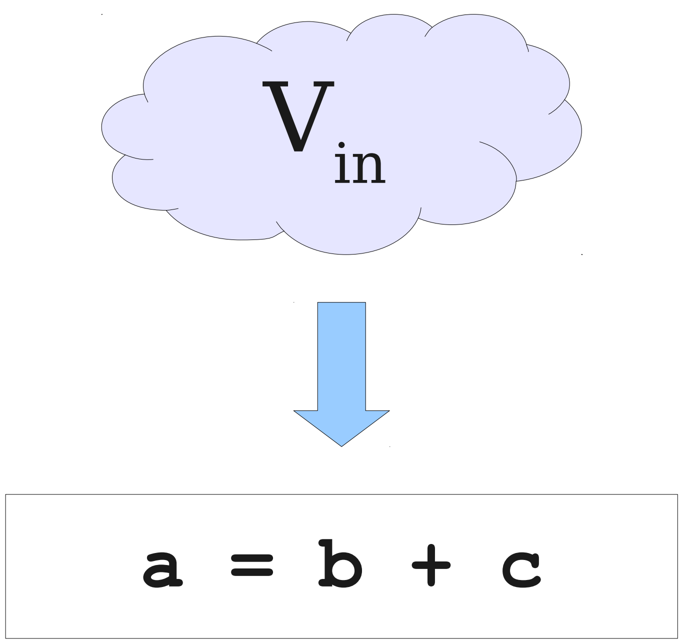

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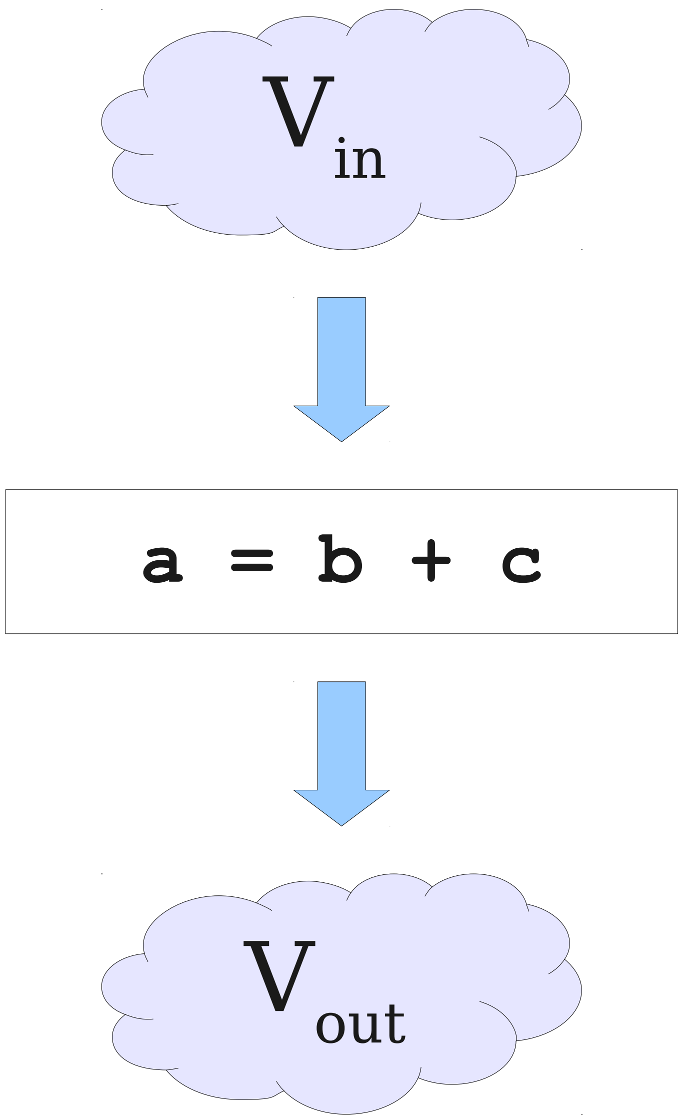

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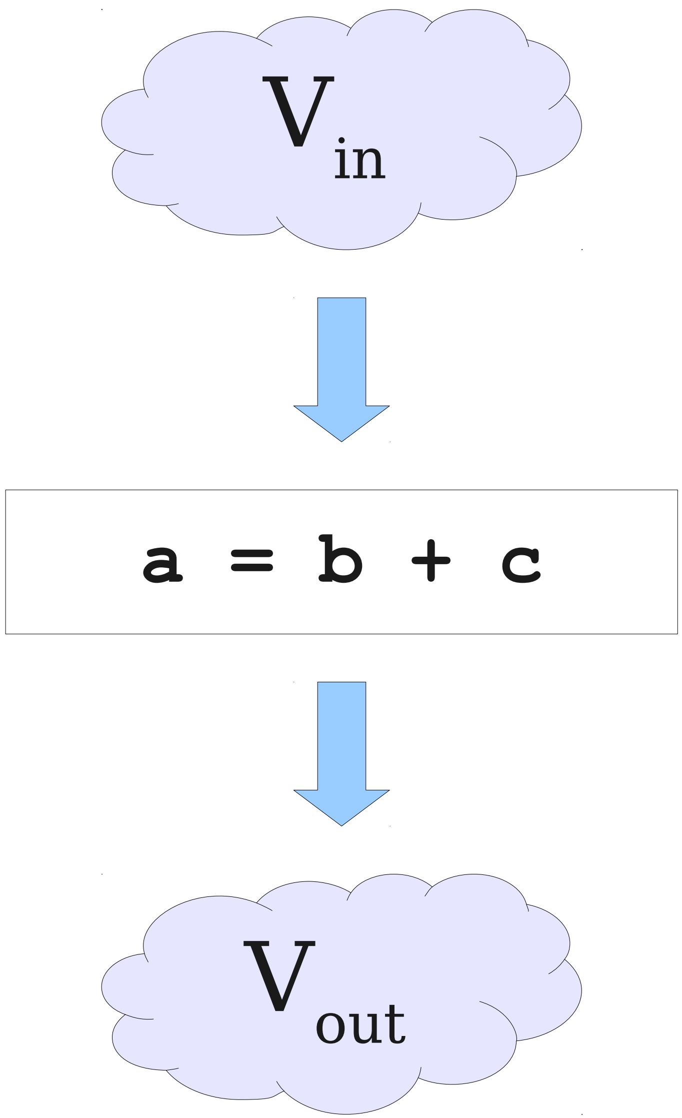

$$V_{out} = f_{a\,=\,b+c}(V_{in})$$

Another View of Local Optimization

- In local optimization, we want to reason about some property of the runtime behavior of the program.
- Could we run the program and just watch what happens?
- **Idea**: Redefine the semantics of our programming language to give us information about our analysis.

Properties of Local Analysis

- The only way to find out what a program will actually do is to run it.
- Problems:
  - The program might not terminate.
  - The program might have some behavior we didn't see when we ran it on a particular input.
- However, this is **not** a problem inside a basic block.
  - Basic blocks contain no loops.
  - There is only one path through the basic block.

Assigning New Semantics

- Example: Available Expressions
- Redefine the statement **a = b + c** to mean "**a** now holds the value of **b + c**, and any variable holding the value **a** is now invalid."
- Run the program assuming these new semantics.
- Treat the optimizer as an interpreter for these new semantics.

Information for a Local Analysis

- What direction are we going?
  - Sometimes forward (available expressions)
  - Sometimes backward (liveness analysis)
- How do we update information after processing a statement?
  - What are the new semantics?
- What information do we know initially?

Formalizing Local Analyses

- Define an analysis of a basic block as a quadruple (D, V, F, I) where
  - **D** is a direction (forwards or backwards)
  - **V** is a set of values the program can have at any point.
  - **F** is a family of **transfer functions** defining the meaning of any expression as a function $f : \mathbf{V} \to \mathbf{V}$.
  - **I** is the initial information at the top (or bottom) of a basic block.

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Available Expressions

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c = b;
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d = a + b;

e = a + b;

d = b;

f = a + b;
```

Available Expressions

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Available Expressions

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Available Expressions

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{ a = b, c = b, d = b, e = a + b }
f = a + b;
```

Available Expressions

{ }

`a = b;`

{ a = b }

`c = b;`

{ a = b, c = b }

`d = a + b;`

{ a = b, c = b, d = a + b }

`e = a + b;`

{ a = b, c = b, d = a + b, e = a + b }

`d = b;`

{ a = b, c = b, d = b, e = a + b }

`f = a + b;`

{ a = b, c = b, d = b, e = a + b, f = a + b }

Available Expressions

- **Direction**: Forward
- **Domain**: Sets of expressions assigned to variables.
- **Transfer functions**: Given a set of variable assignments V and statement **a = b + c**:
  - Remove from V any expression containing **a** as a subexpression.
  - Add to V the expression **a = b + c**.
- **Initial value**: Empty set of expressions.

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;

f = e;
```

{ b, d }

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;

d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;

e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;

d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
a = b;

c = a;
```
`{ a, b }`
```
d = a + b;
```
`{ a, b, d }`
```
e = d;
```
`{ a, b, e }`
```
d = a;
```
`{ b, d, e }`
```
f = e;
```
`{ b, d }`

Liveness Analysis

```
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

```
{ b }
a = b;
{ a, b }
c = a;
{ a, b }
d = a + b;
{ a, b, d }
e = d;
{ a, b, e }
d = a;
{ b, d, e }
f = e;
{ b, d }
```

Liveness Analysis

- **Direction**: Backwards
- **Domain**: Sets of variables.
- **Transfer function**: Given a set of variables *V* and statement **a = b + c**:
  - Remove **a** from *V* (any previous value of **a** is now dead.)
  - Add **b** and **c** to *V* (any previous value of **b** or **c** is now live.)
  - Formally: $f_{a = b + c}(V) = (V - \{\mathbf{a}\}) \cup \{\mathbf{b}, \mathbf{c}\}$
- **Initial value**: Depends on semantics of language.

Running Local Analyses

- Given an analysis (**D**, **V**, **F**, **I**) for a basic block.
  - Assume that **D** is “forward;” analogous for the reverse case.
- Initially, set OUT[**entry**] to **I**.
- For each statement **s**, in order:
  - Set IN[**s**] to OUT[**prev**], where **prev** is the previous statement.
  - Set OUT[**s**] to f_s(IN[**s**]), where f_s is the transfer function for statement **s**.

Global Optimizations

Global Analysis

- A **global analysis** is an analysis that works on a control-flow graph as a whole.
- Substantially more powerful than a local analysis.
  - (Why?)
- Substantially more complicated than a local analysis.
  - (Why?)

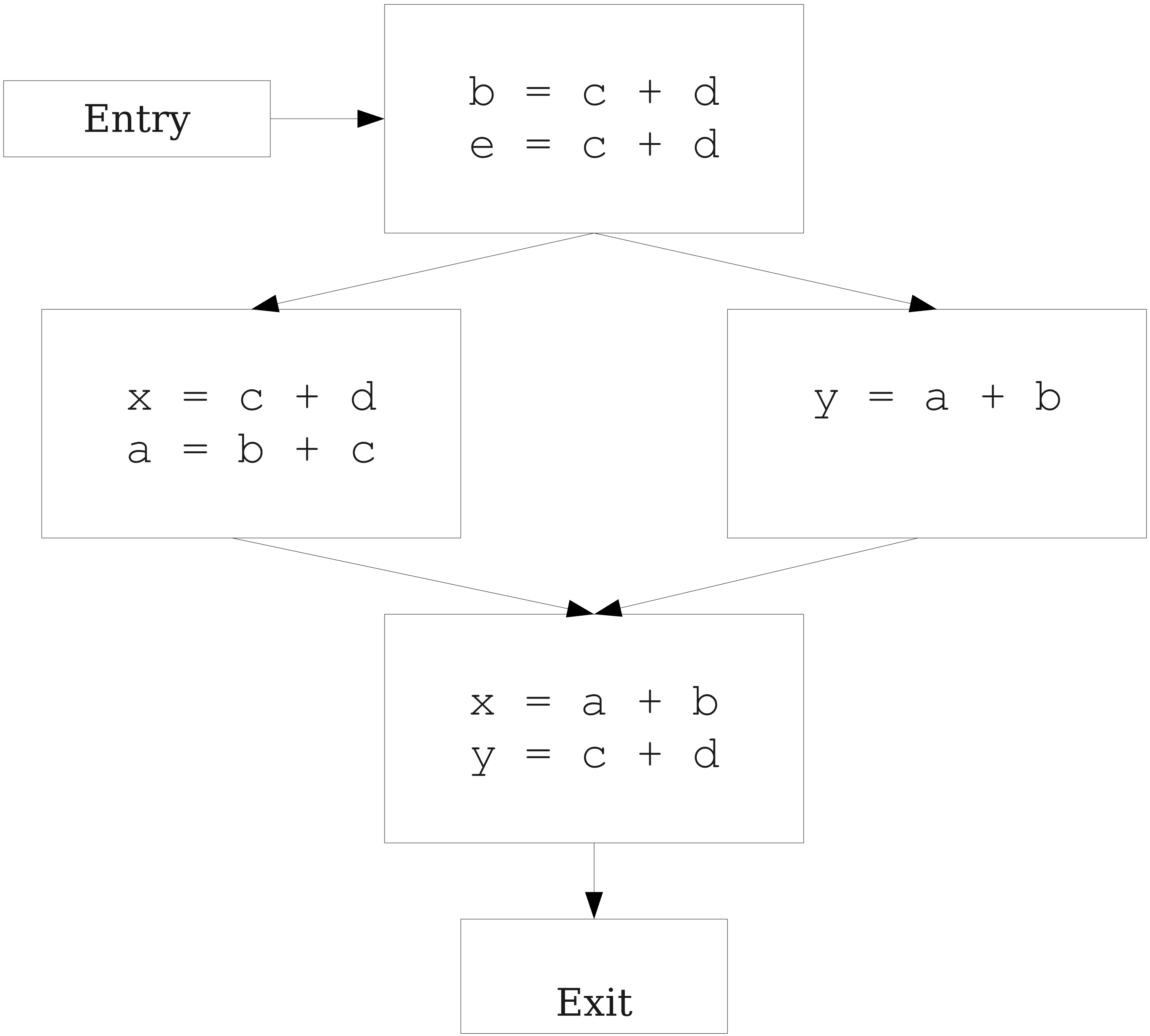
Local vs. Global Analysis

- Many of the optimizations from local analysis can still be applied globally.
  - We'll see how to do this later today.
- Certain optimizations are possible in global analysis that aren't possible locally:
  - e.g. **code motion:** Moving code from one basic block into another to avoid computing values unnecessarily.
- We'll explore three analyses in detail:
  - **Global dead code elimination**.
  - **Global constant propagation**.
  - **Partial redundancy elimination**.

Global Dead Code Elimination

- Local dead code elimination needed to know what variables were live on exit from a basic block.
- This information can only be computed as part of a global analysis.
- How do we modify our liveness analysis to handle a CFG?

CFGs Without Loops



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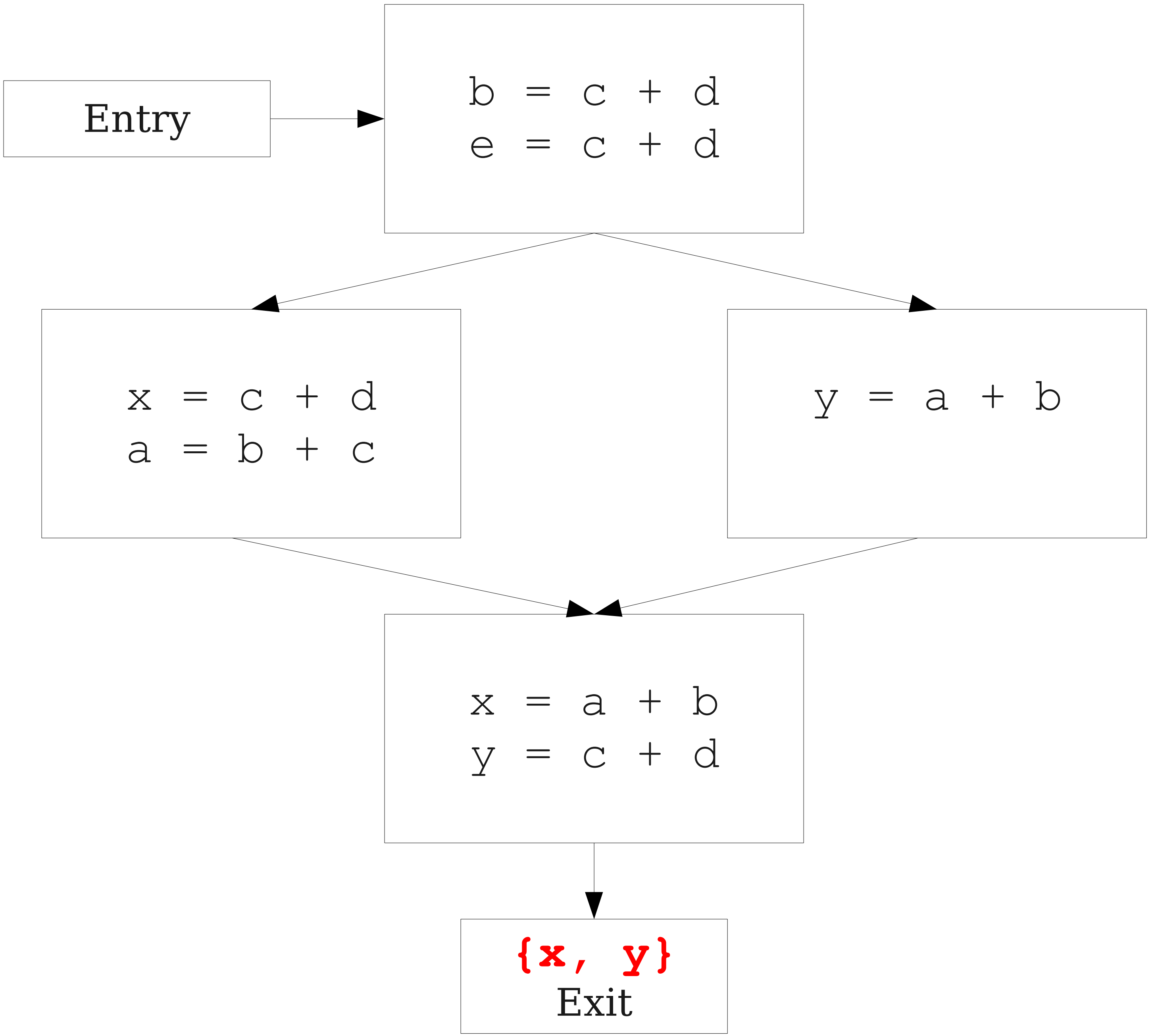

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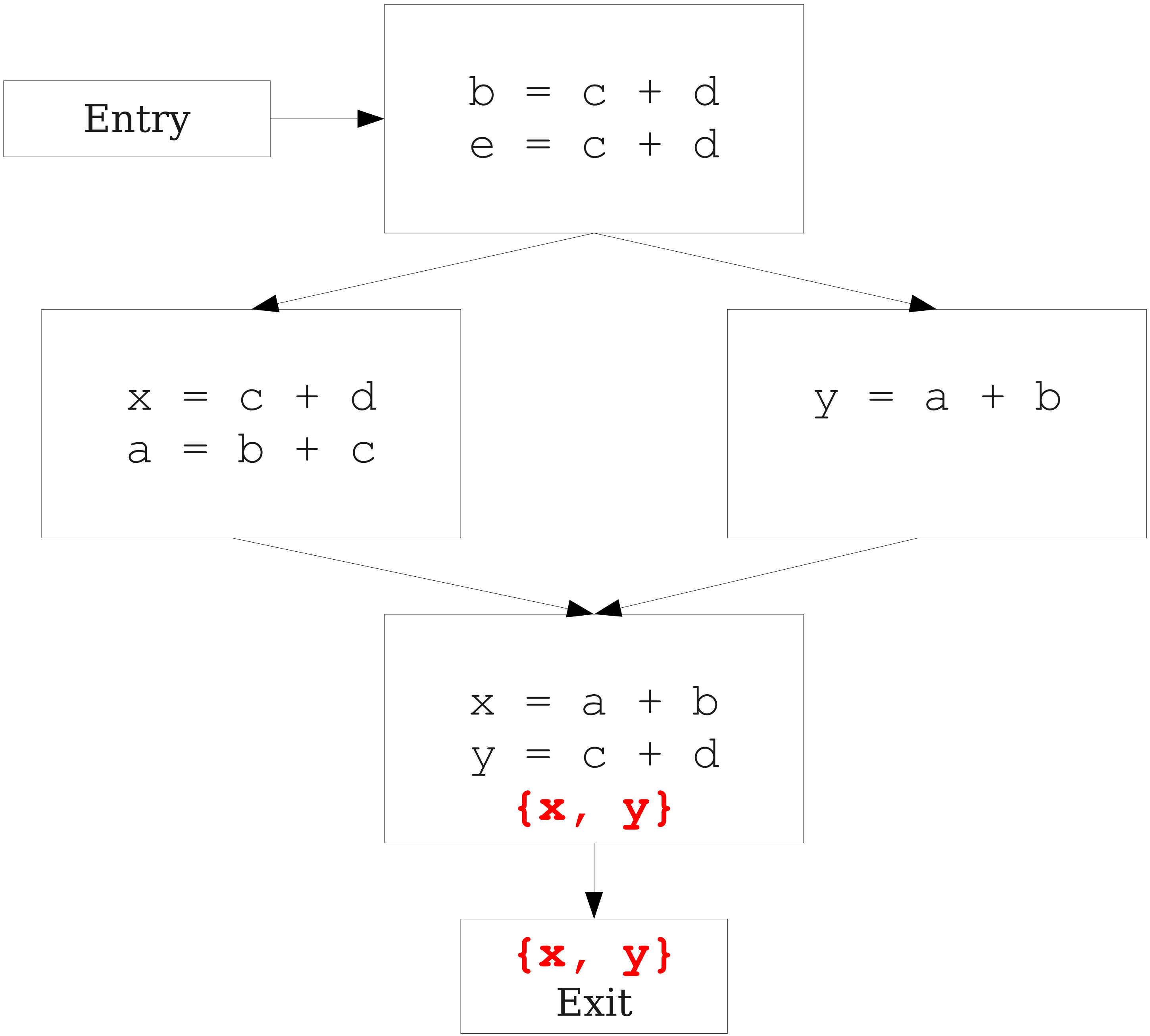

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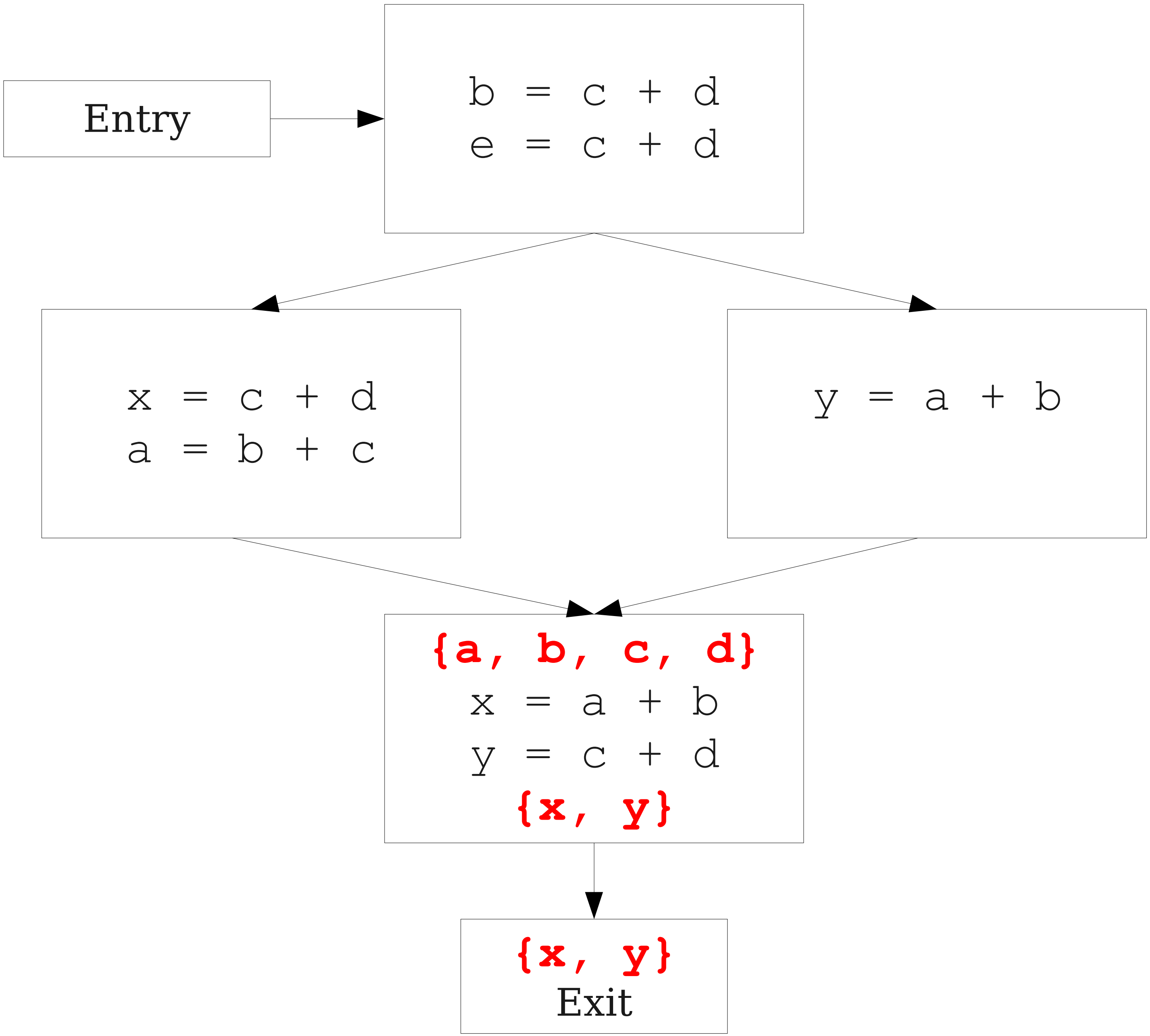

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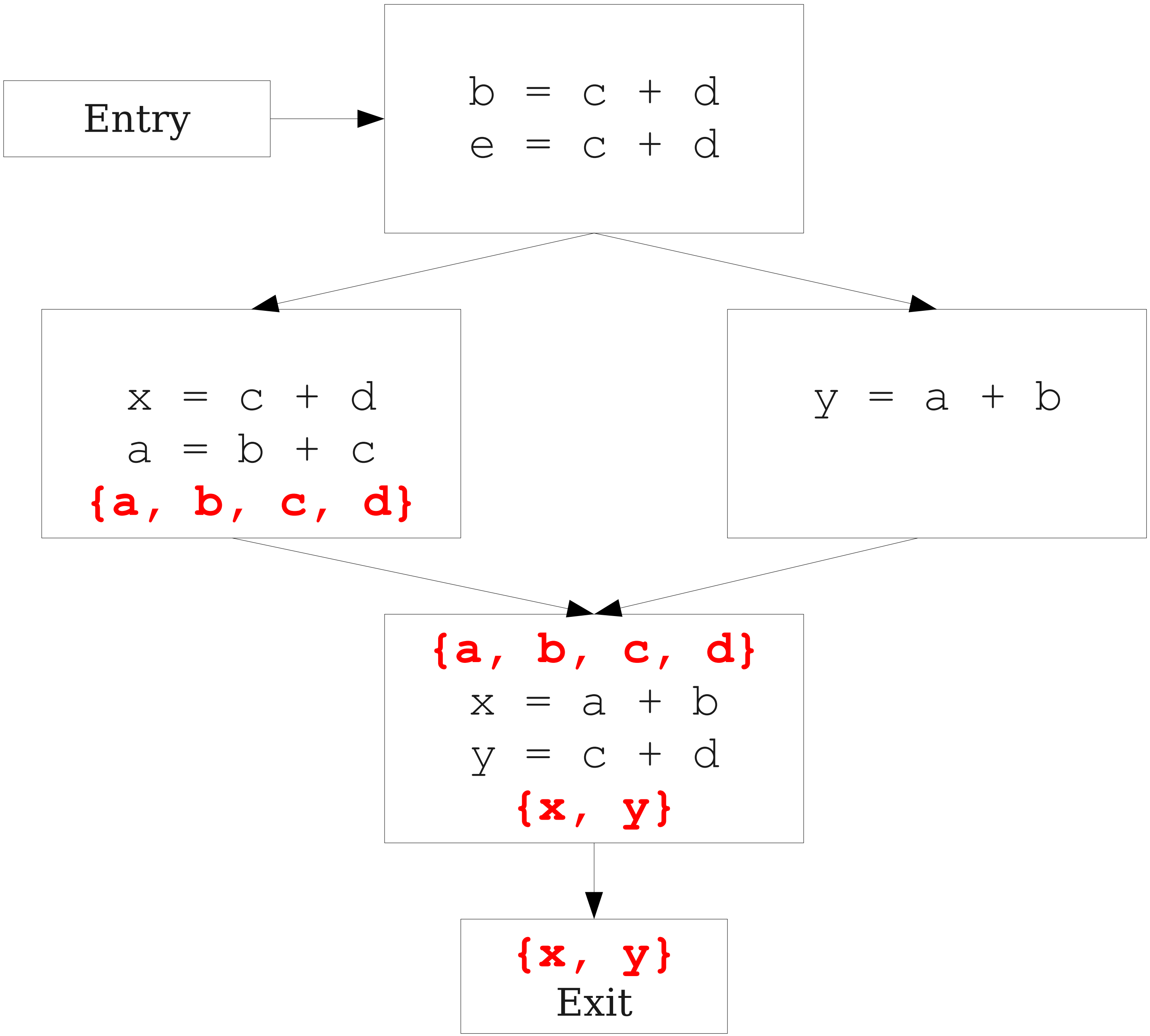

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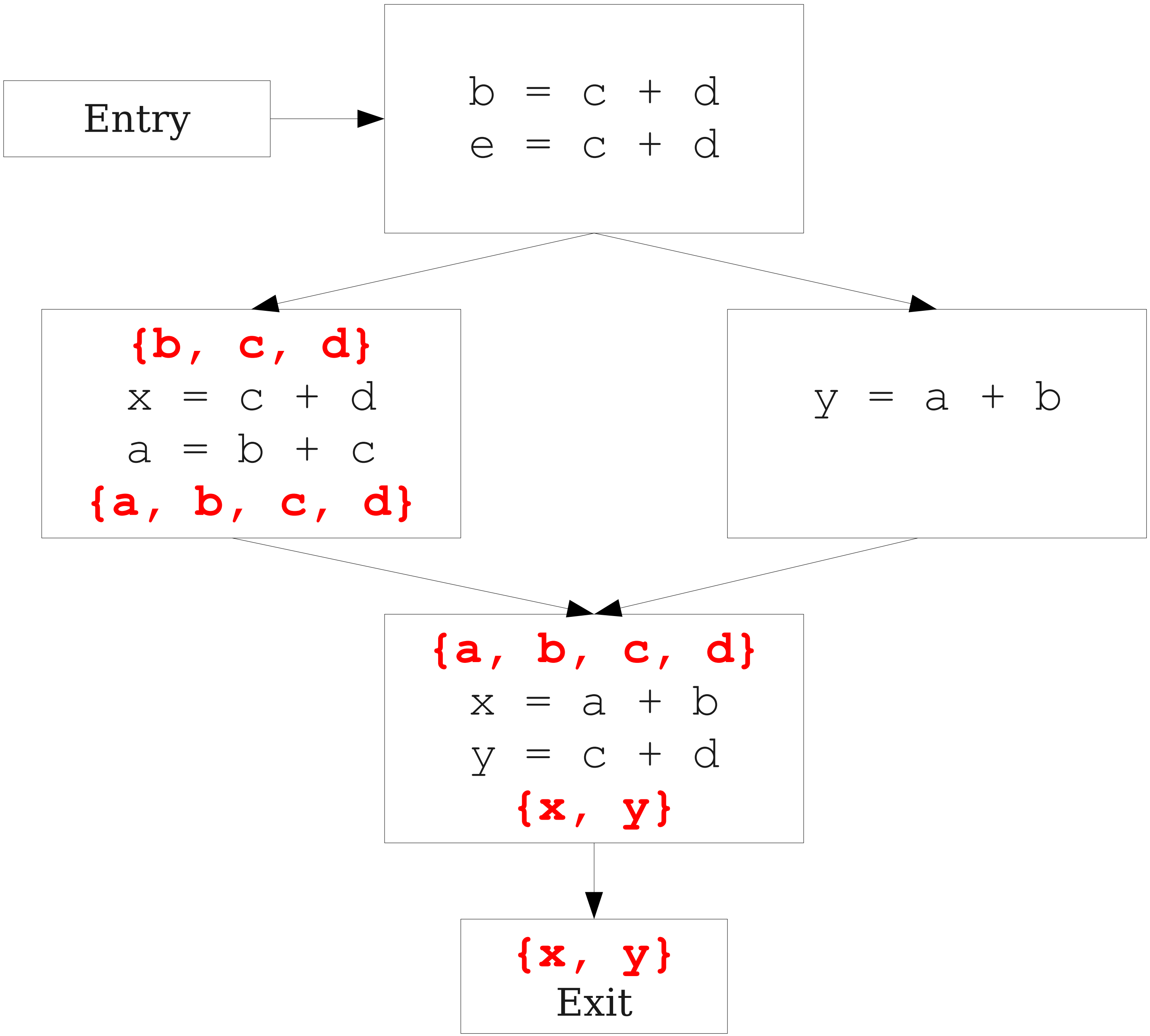

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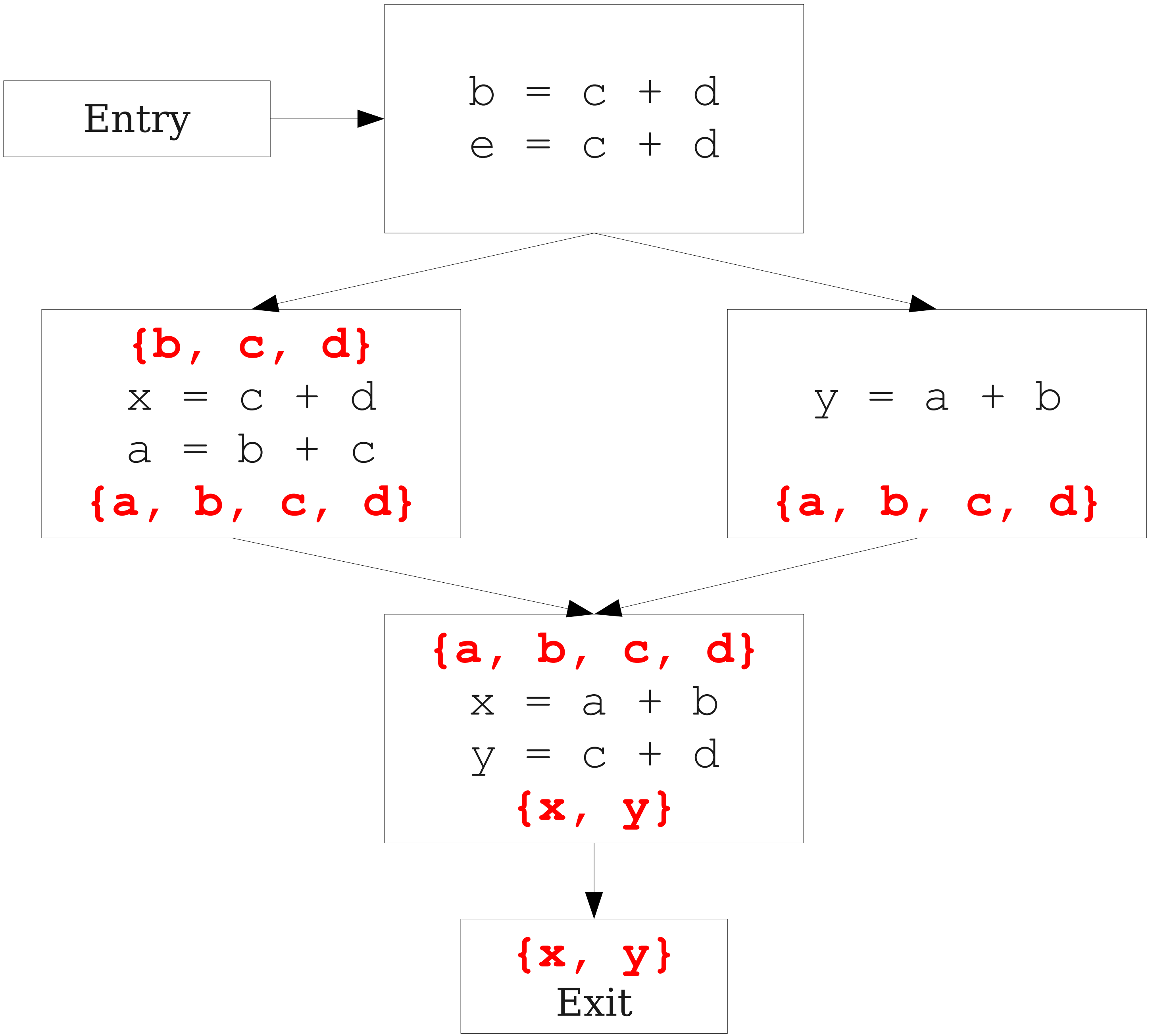

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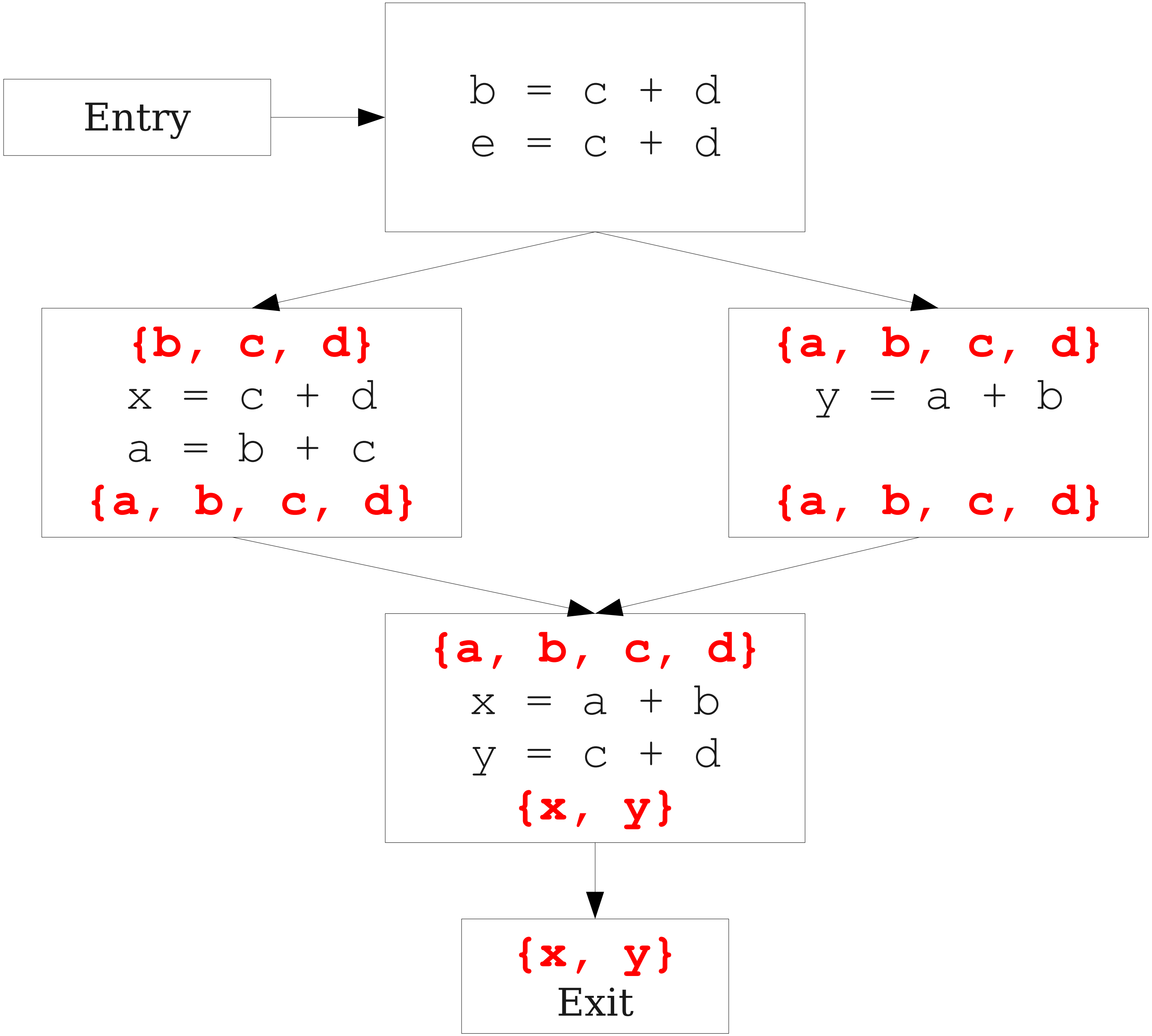

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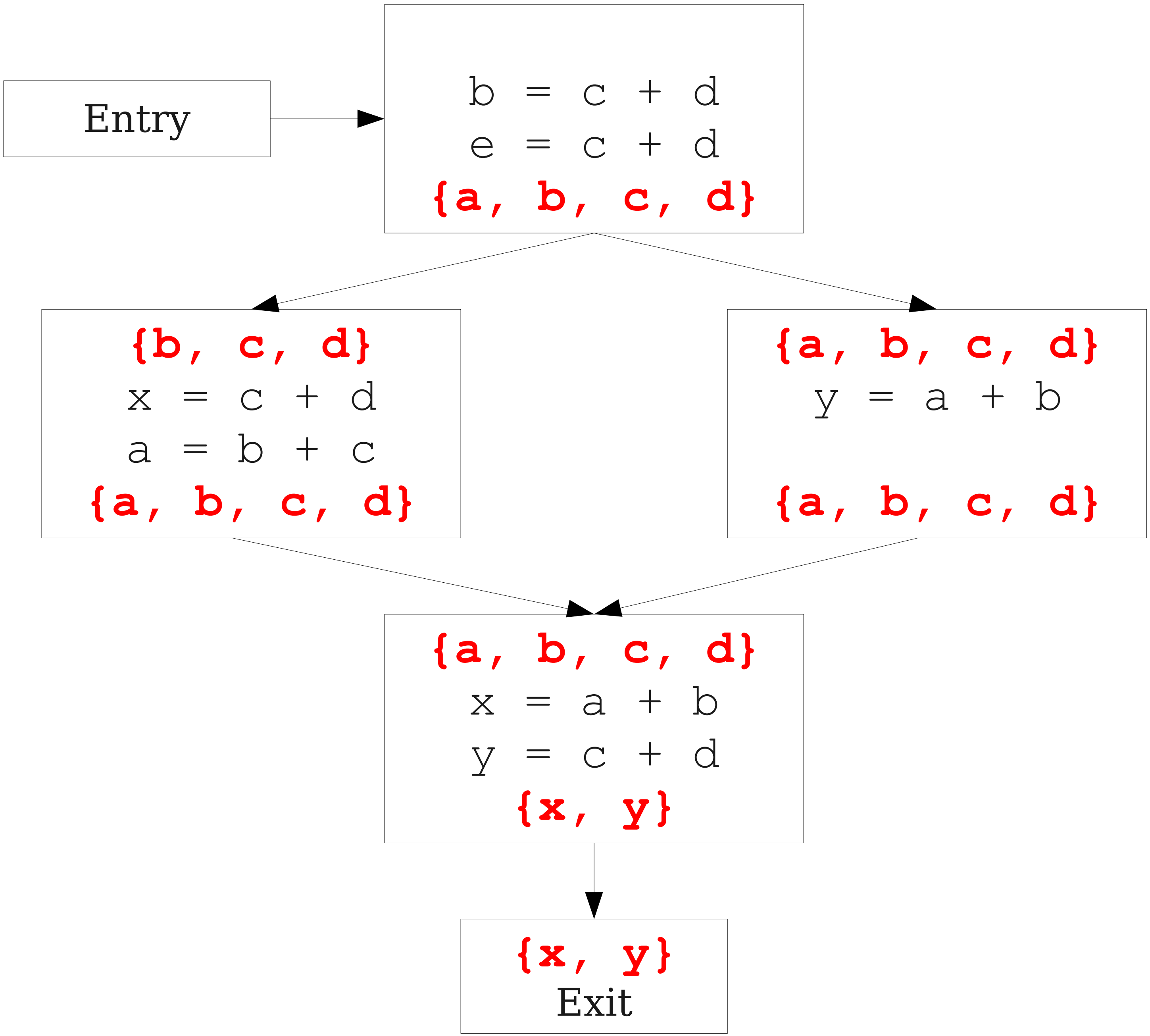

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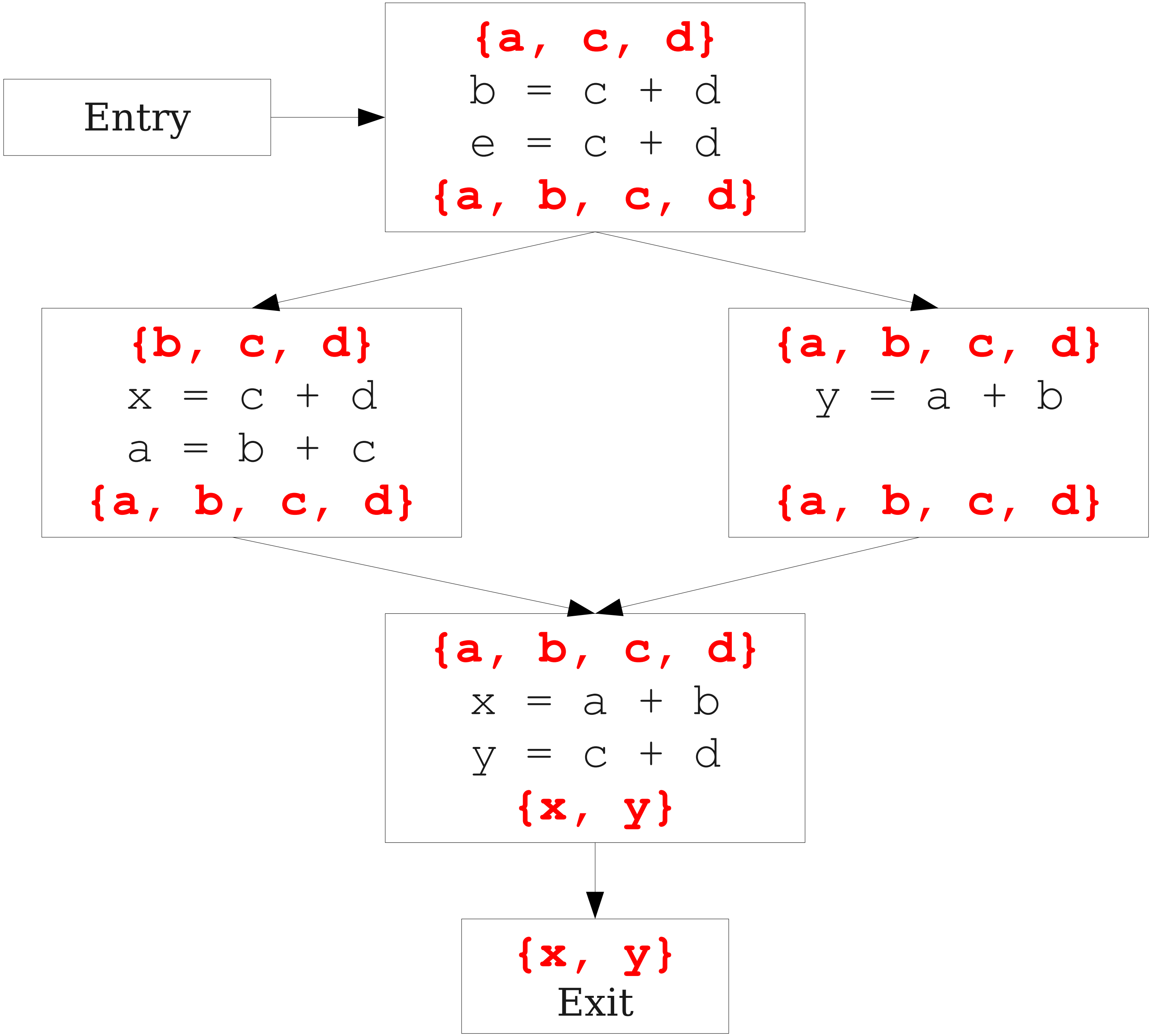

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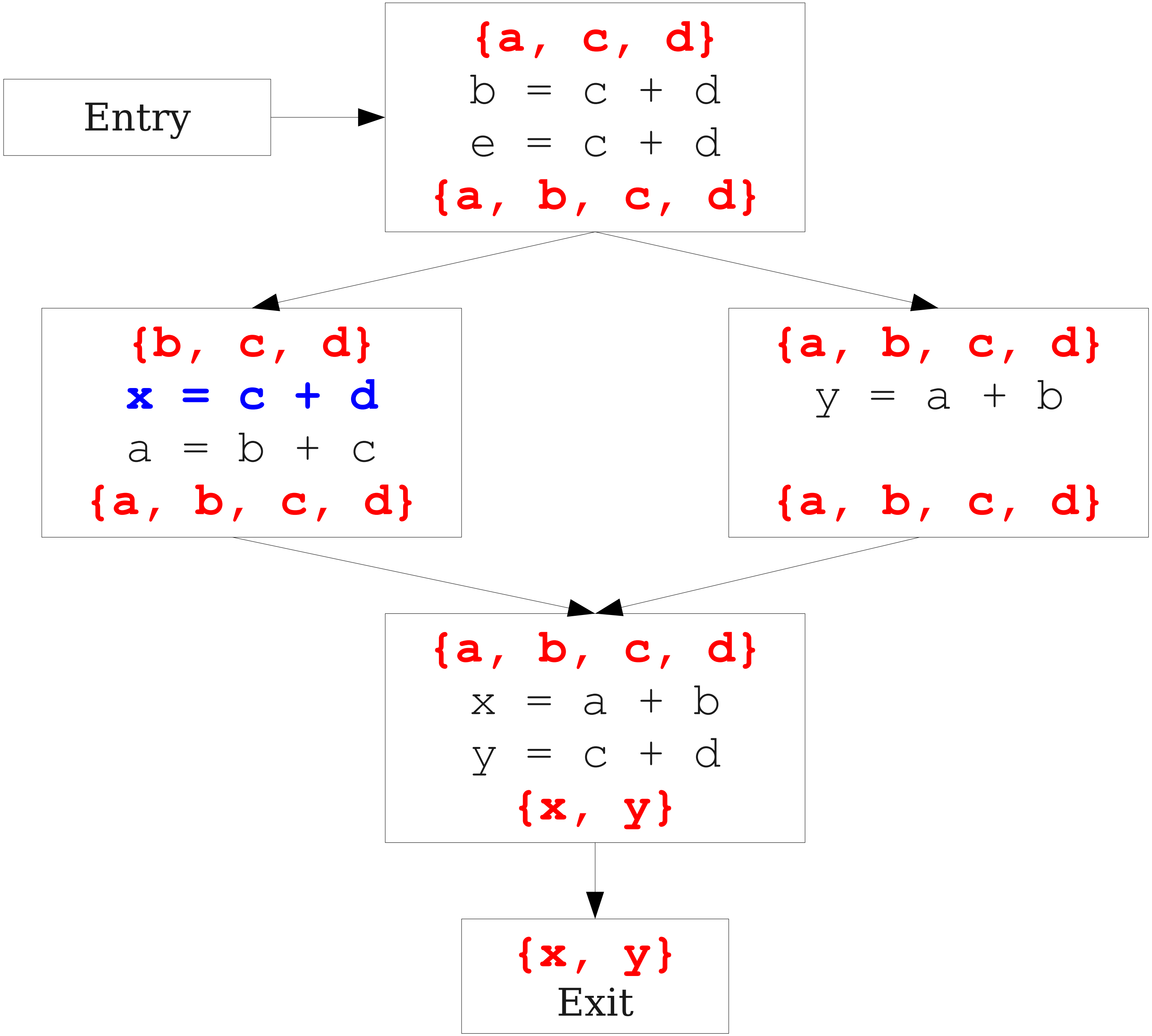

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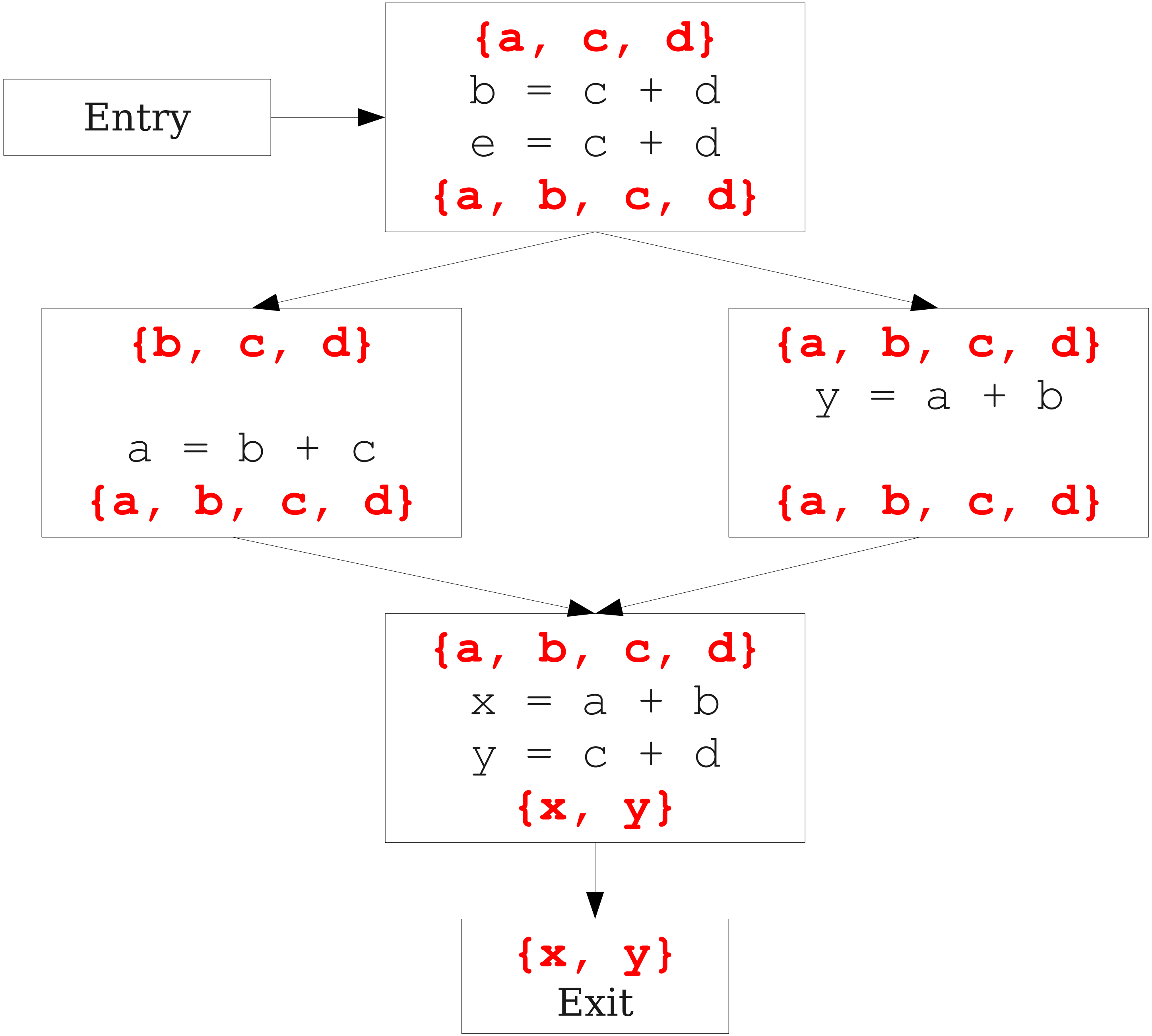

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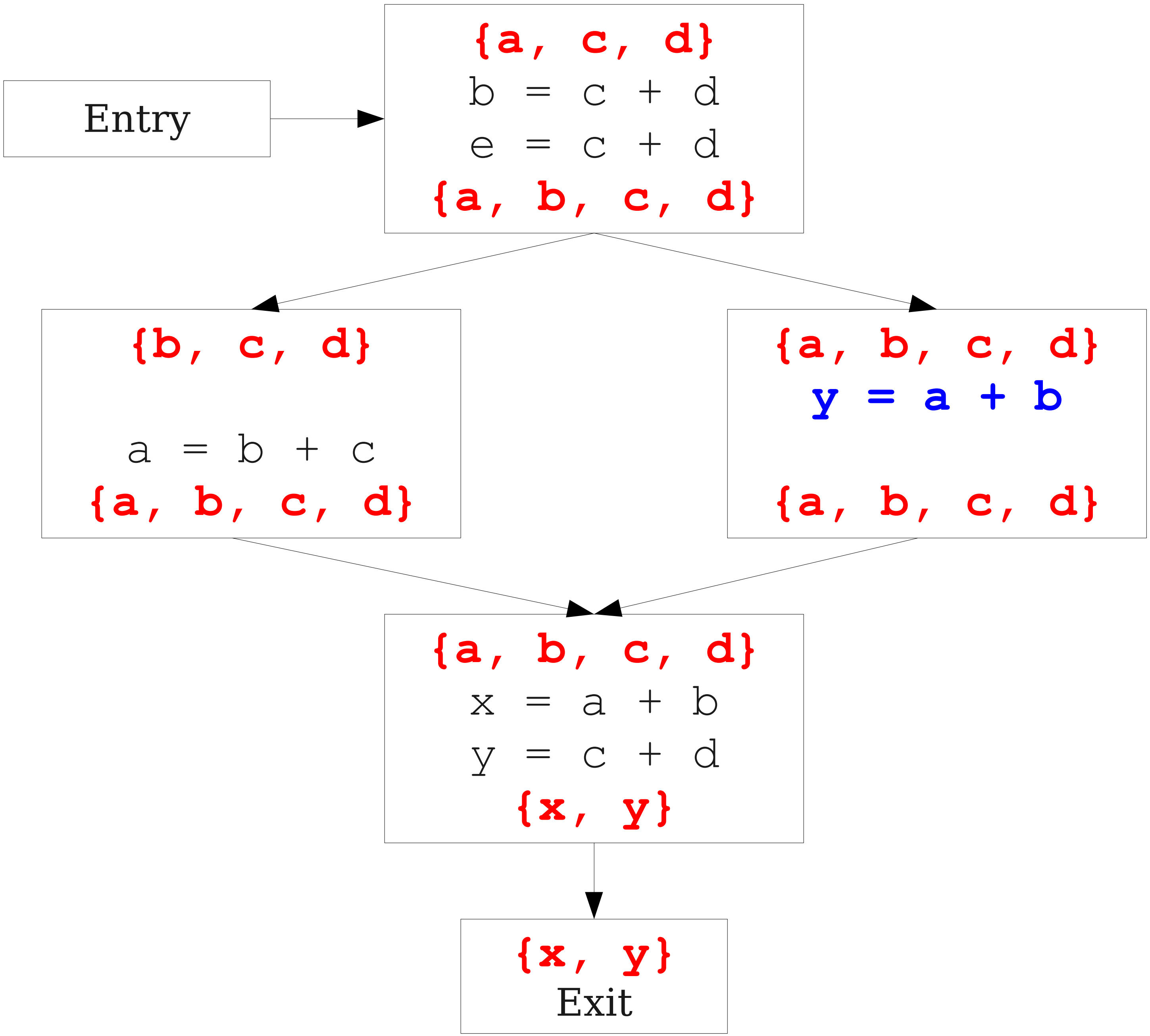

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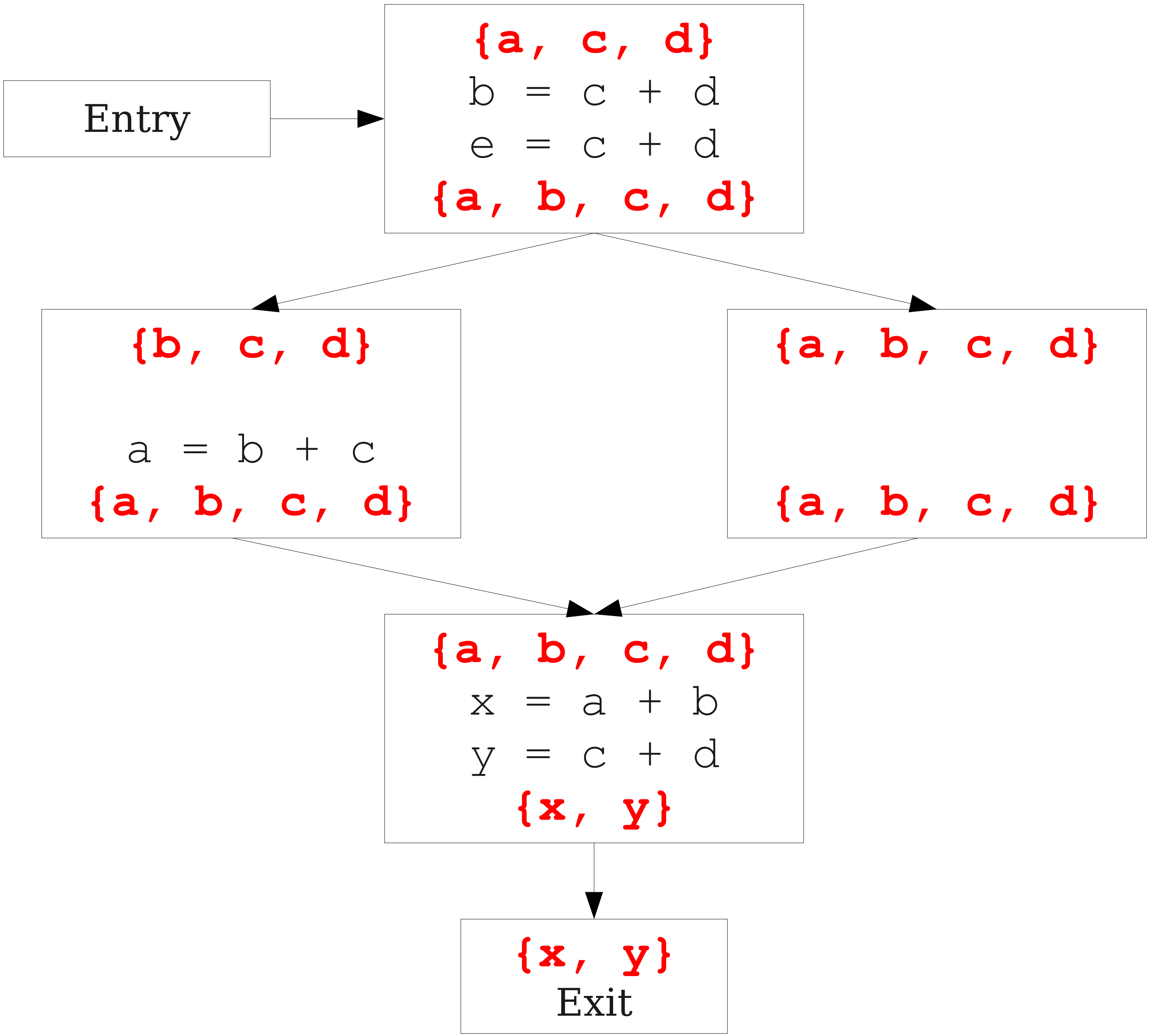

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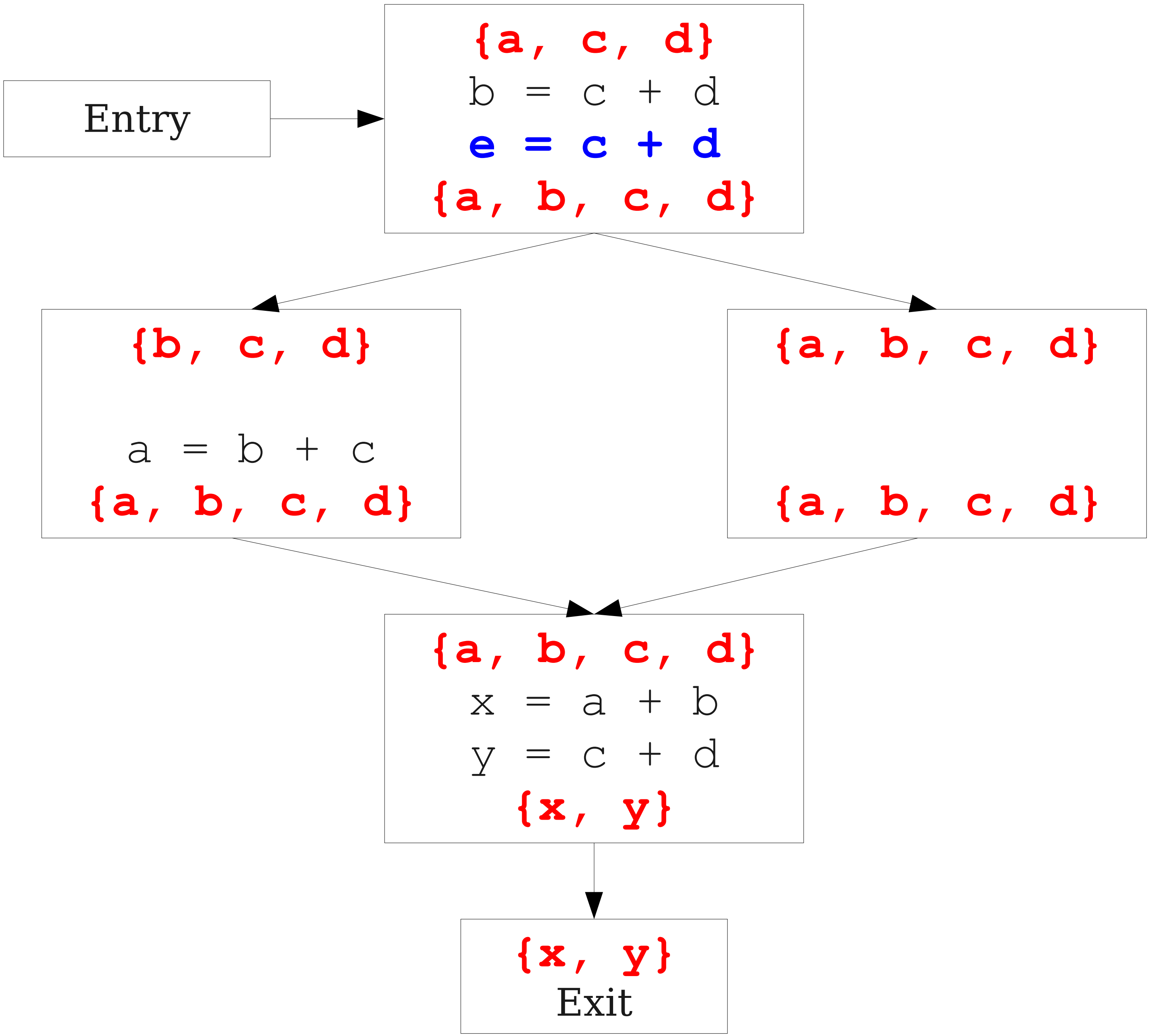

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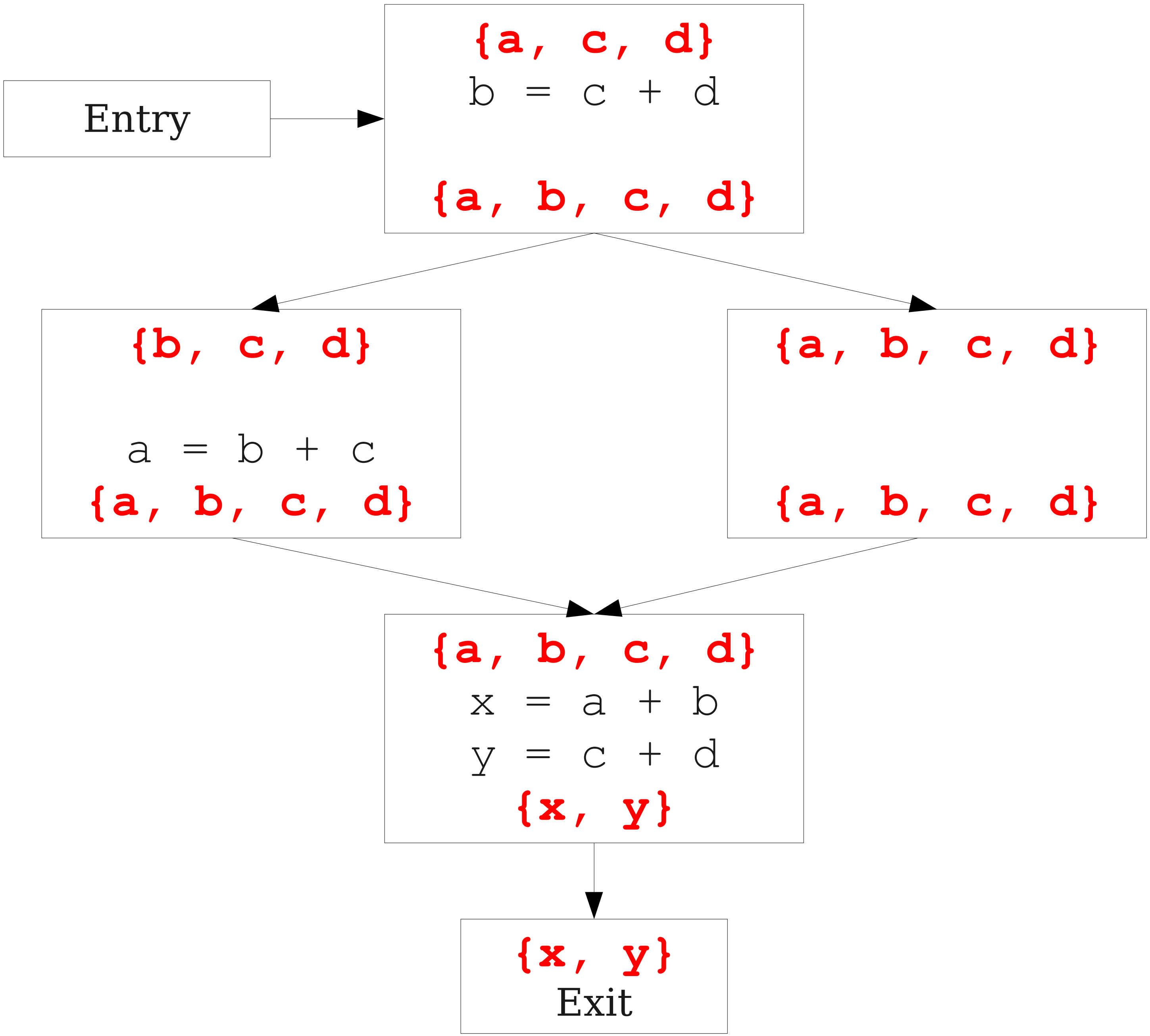

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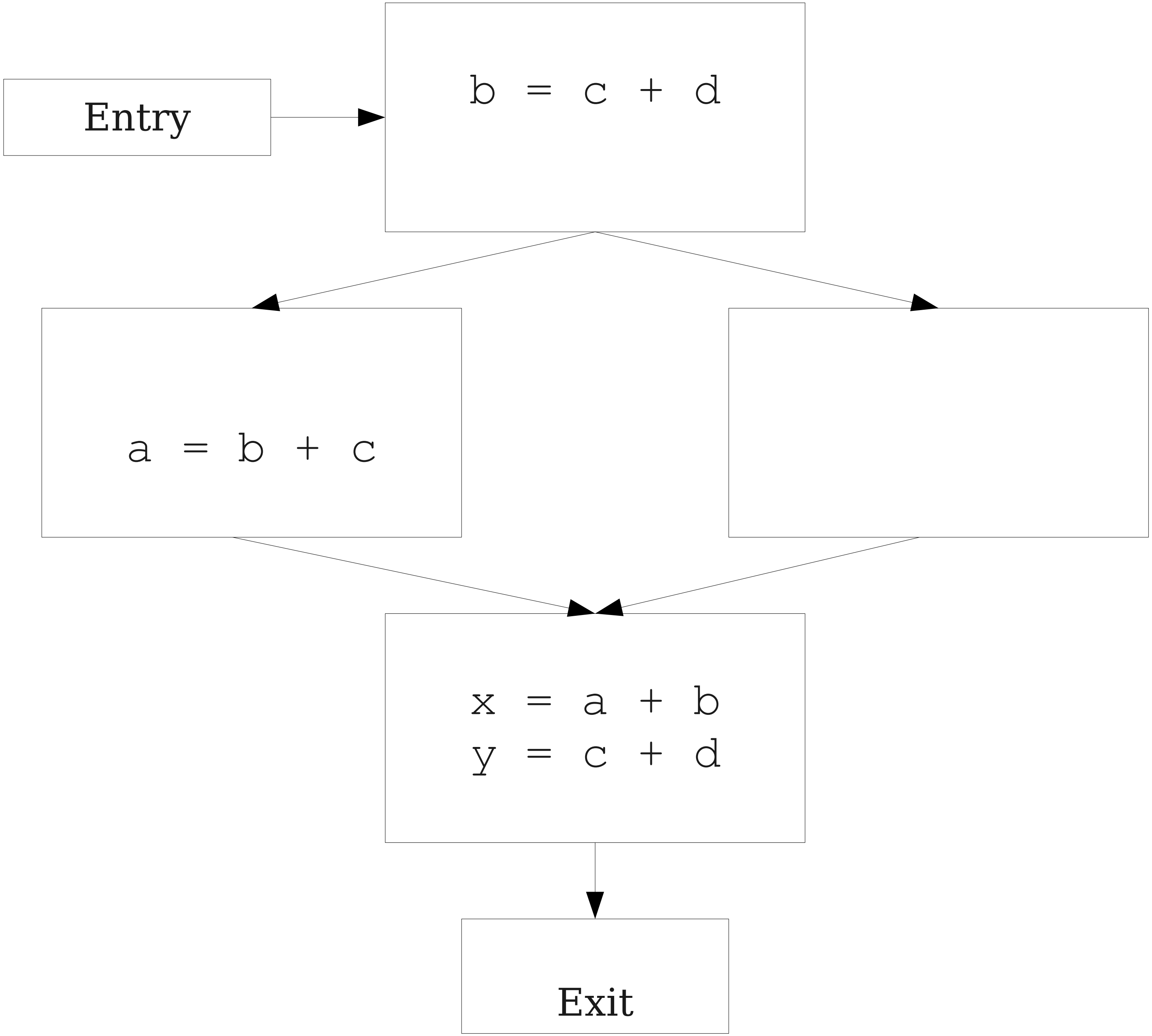

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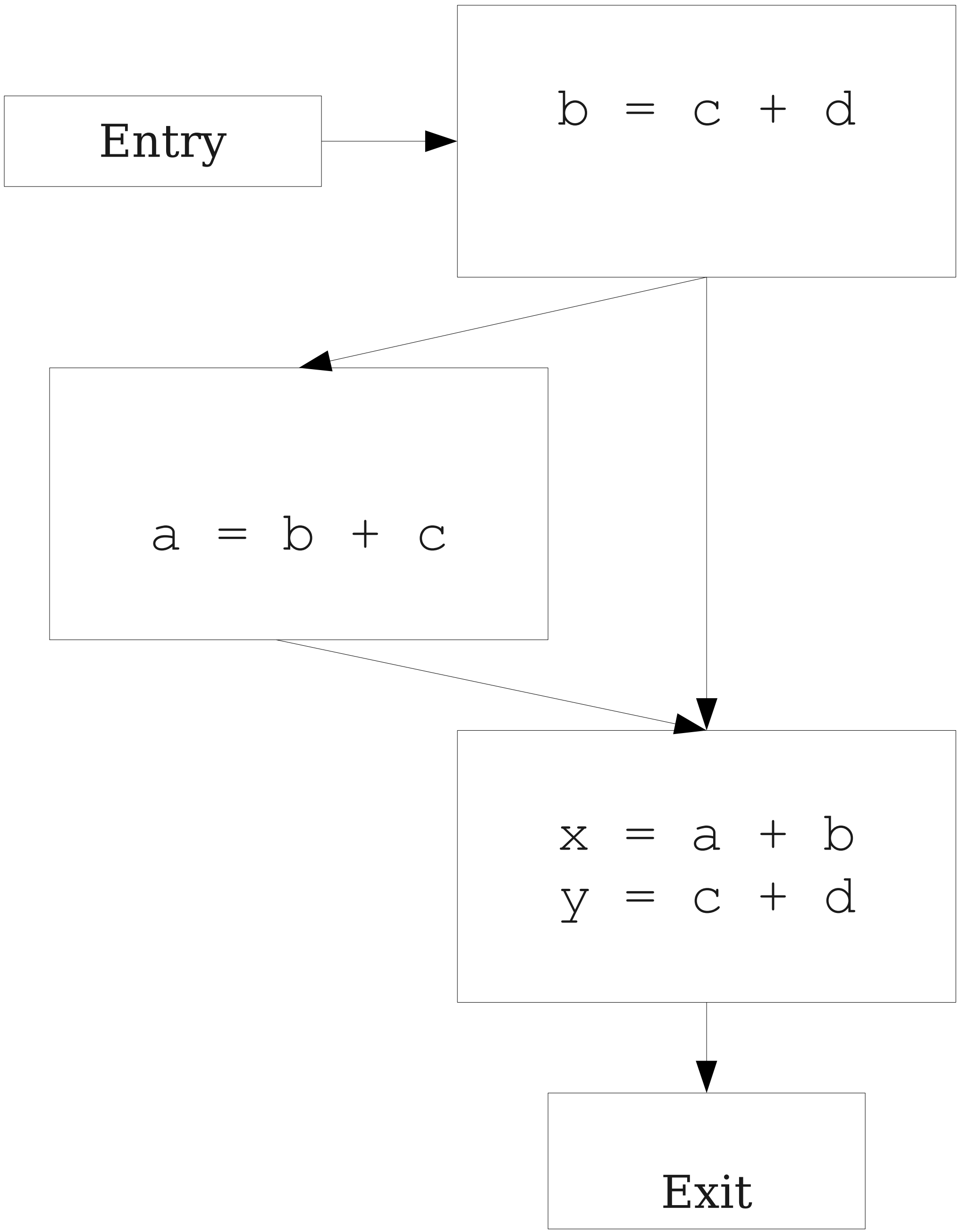
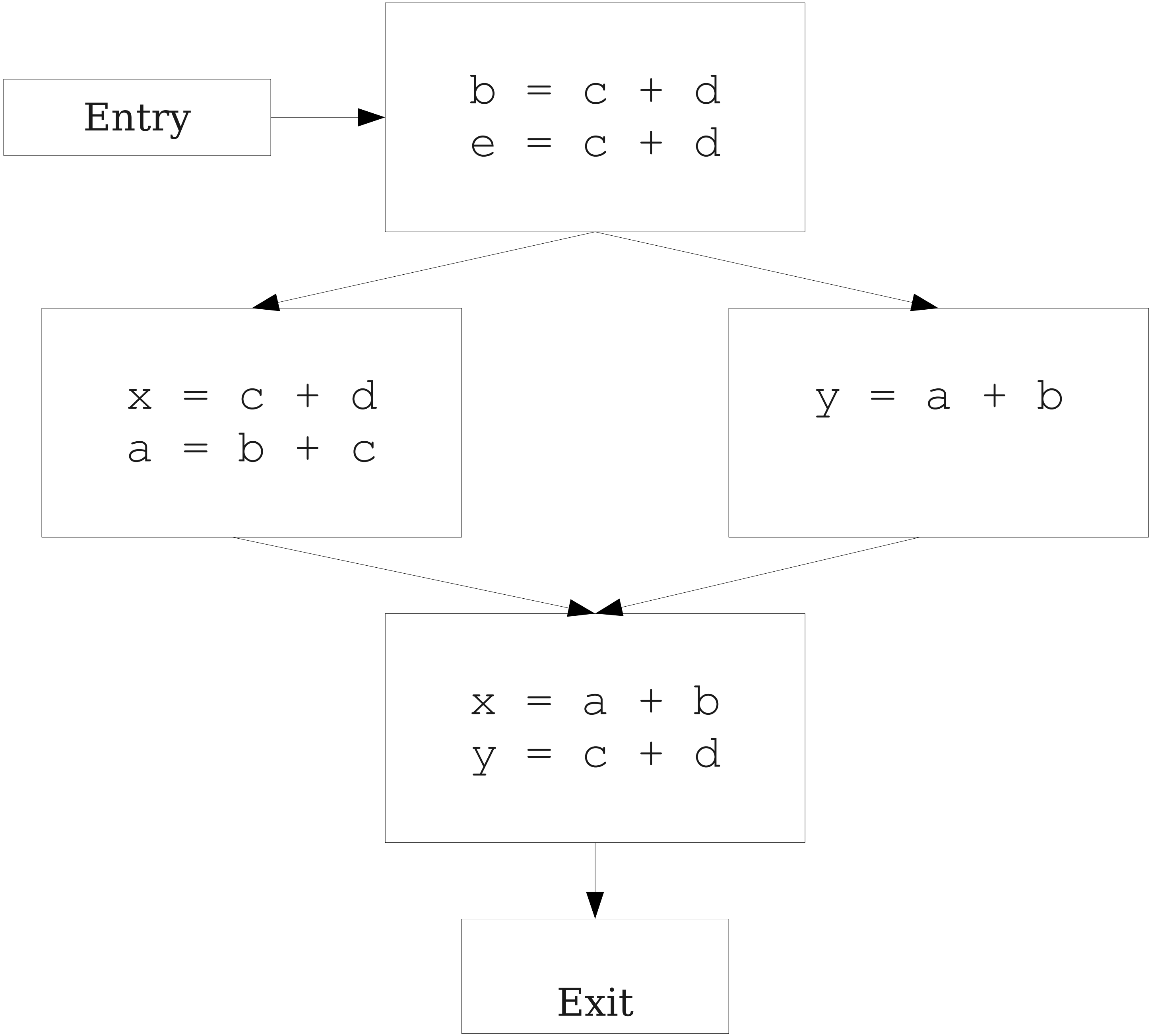


Major Changes, Part One

- In a local analysis, each statement has exactly one predecessor.
- In a global analysis, each statement may have **multiple** predecessors.
- A global analysis must have some means of combining information from all predecessors of a basic block.

CFGs Without Loops



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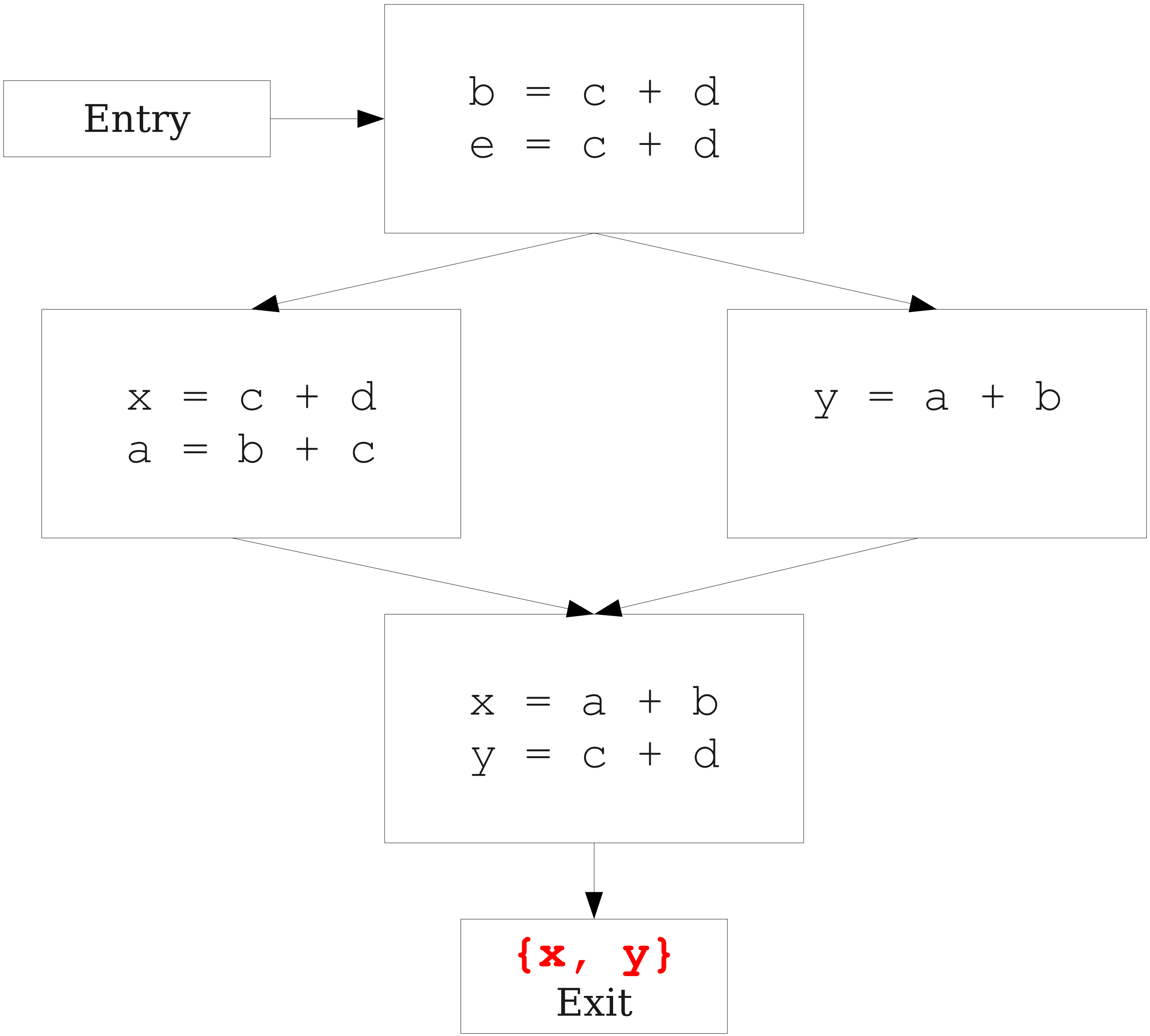

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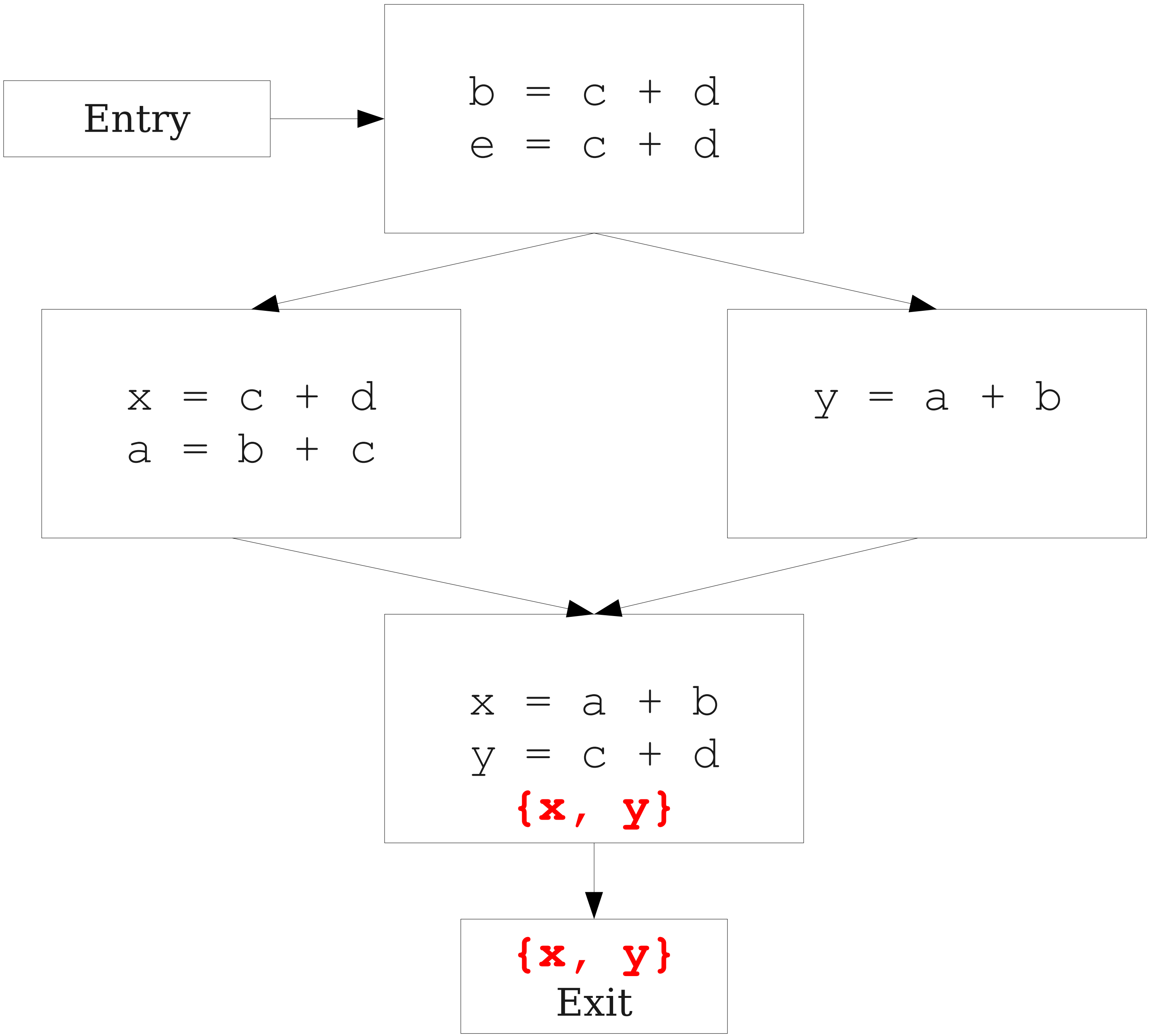

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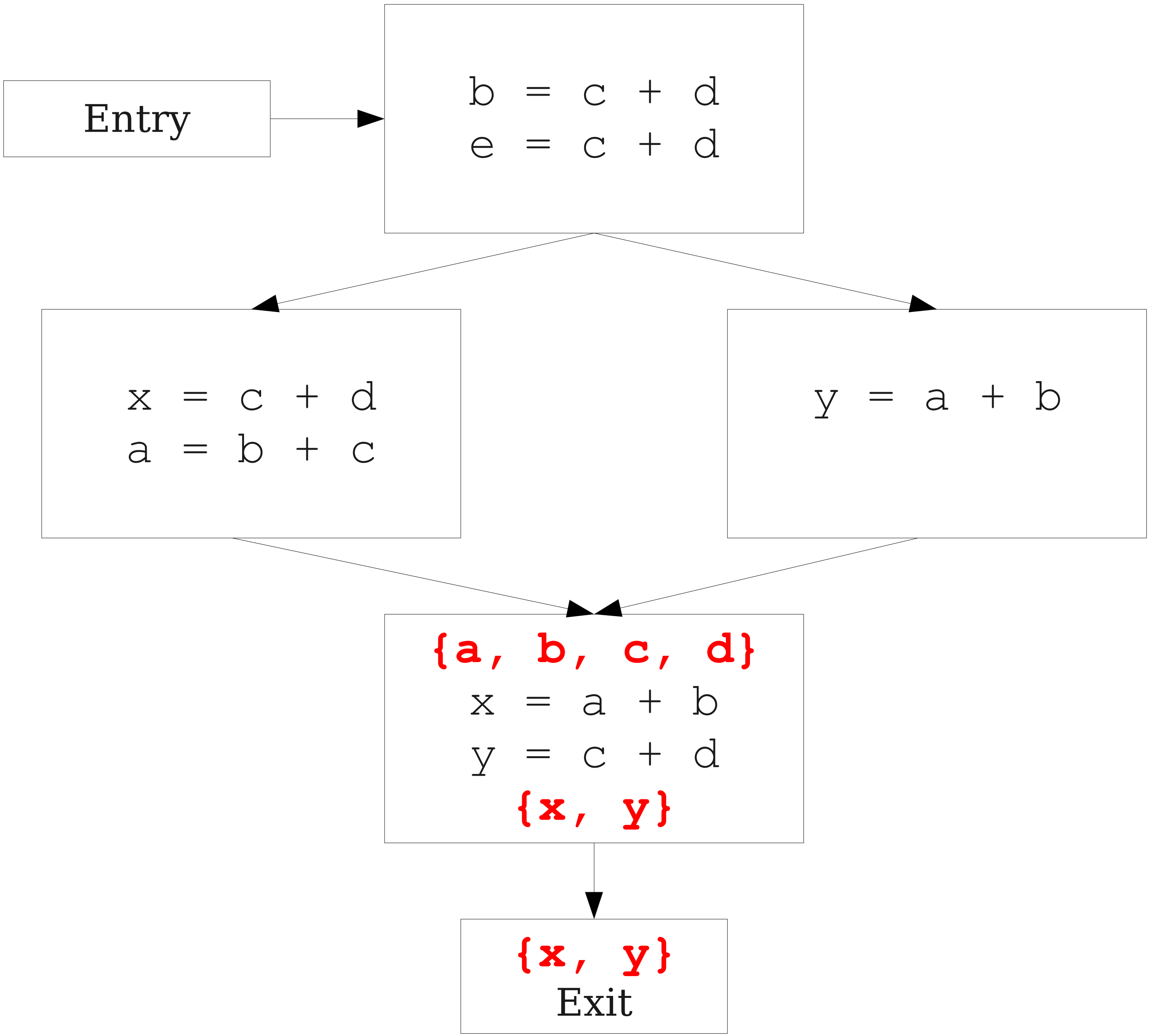

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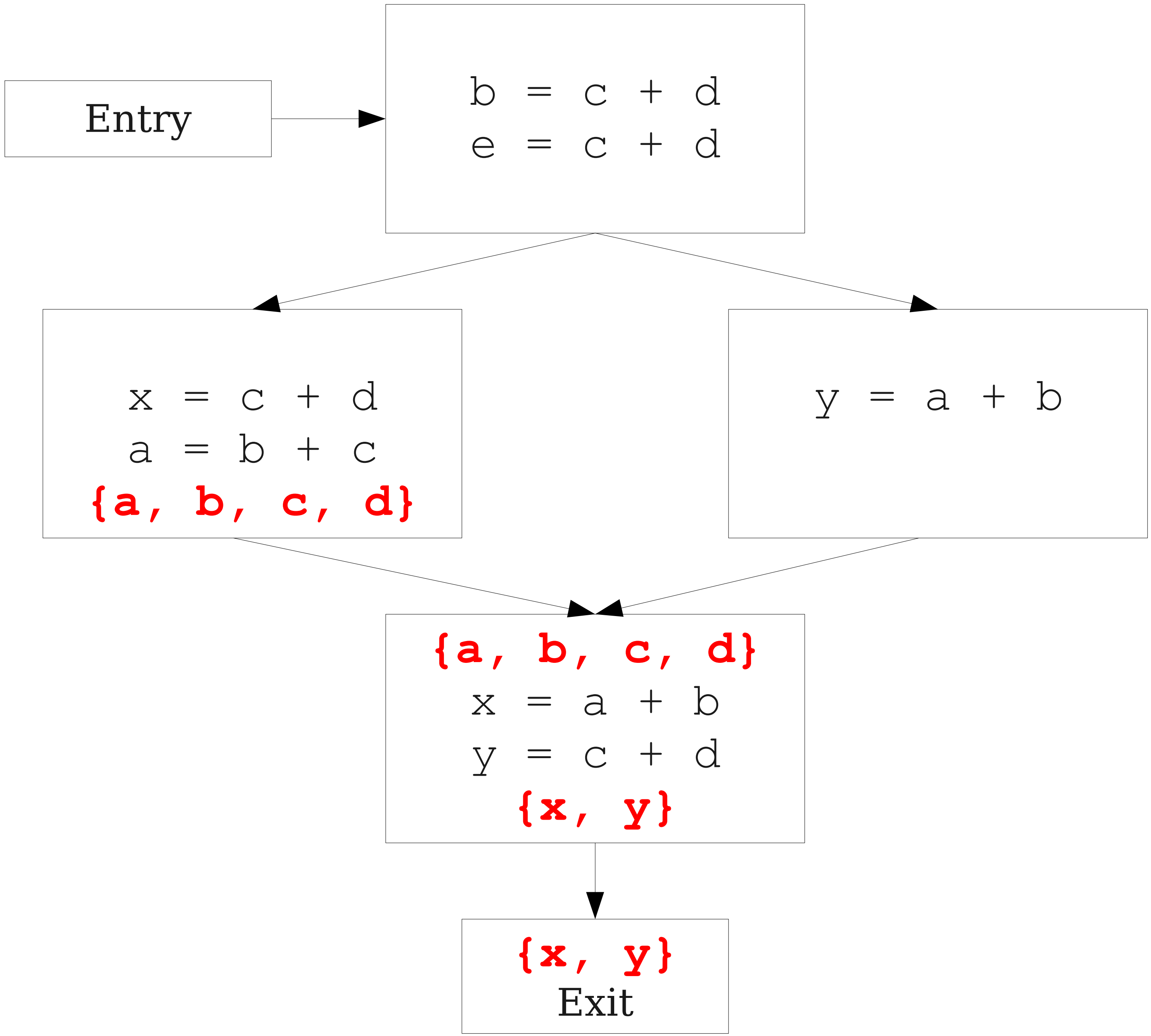

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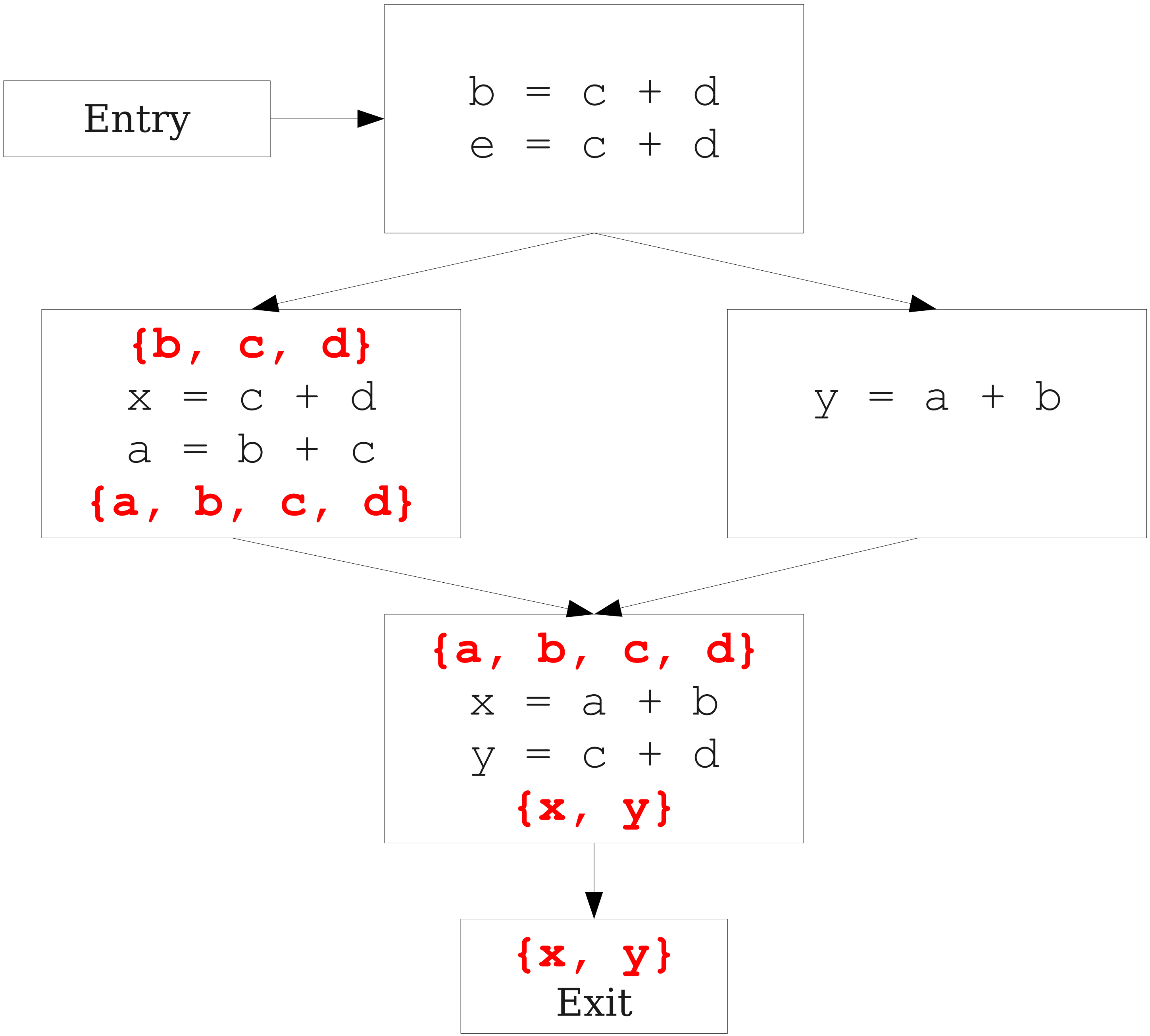

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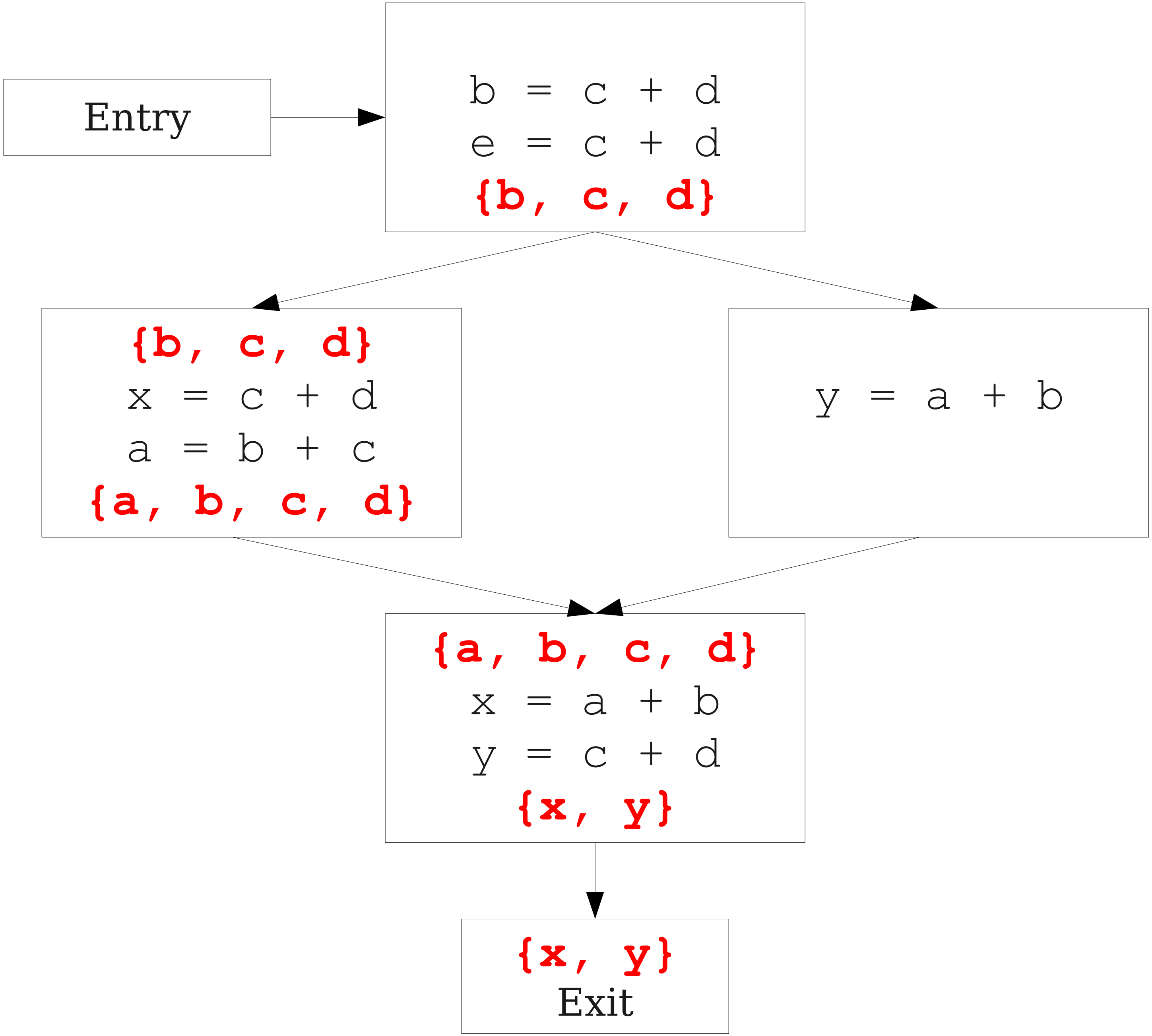

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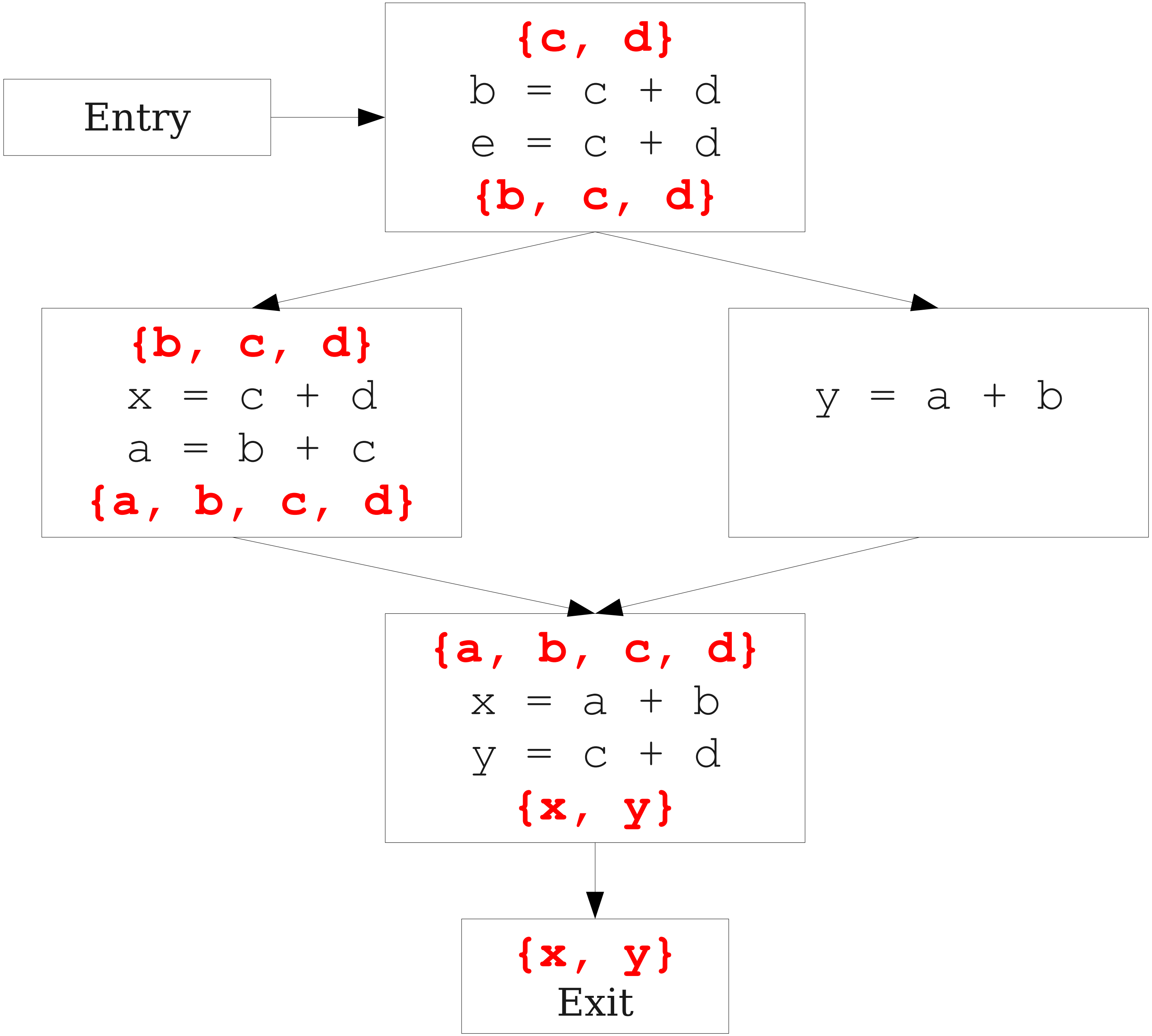

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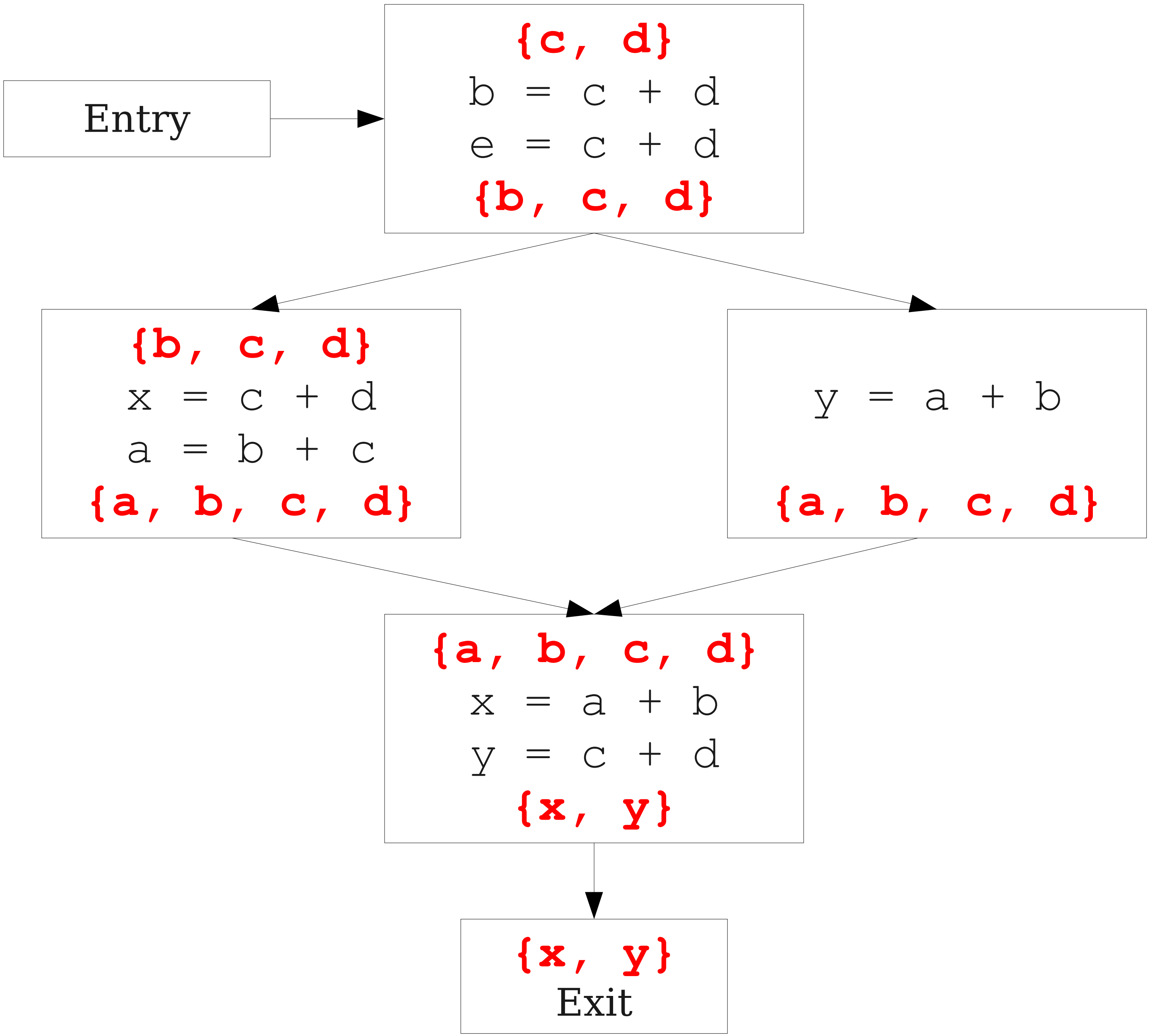

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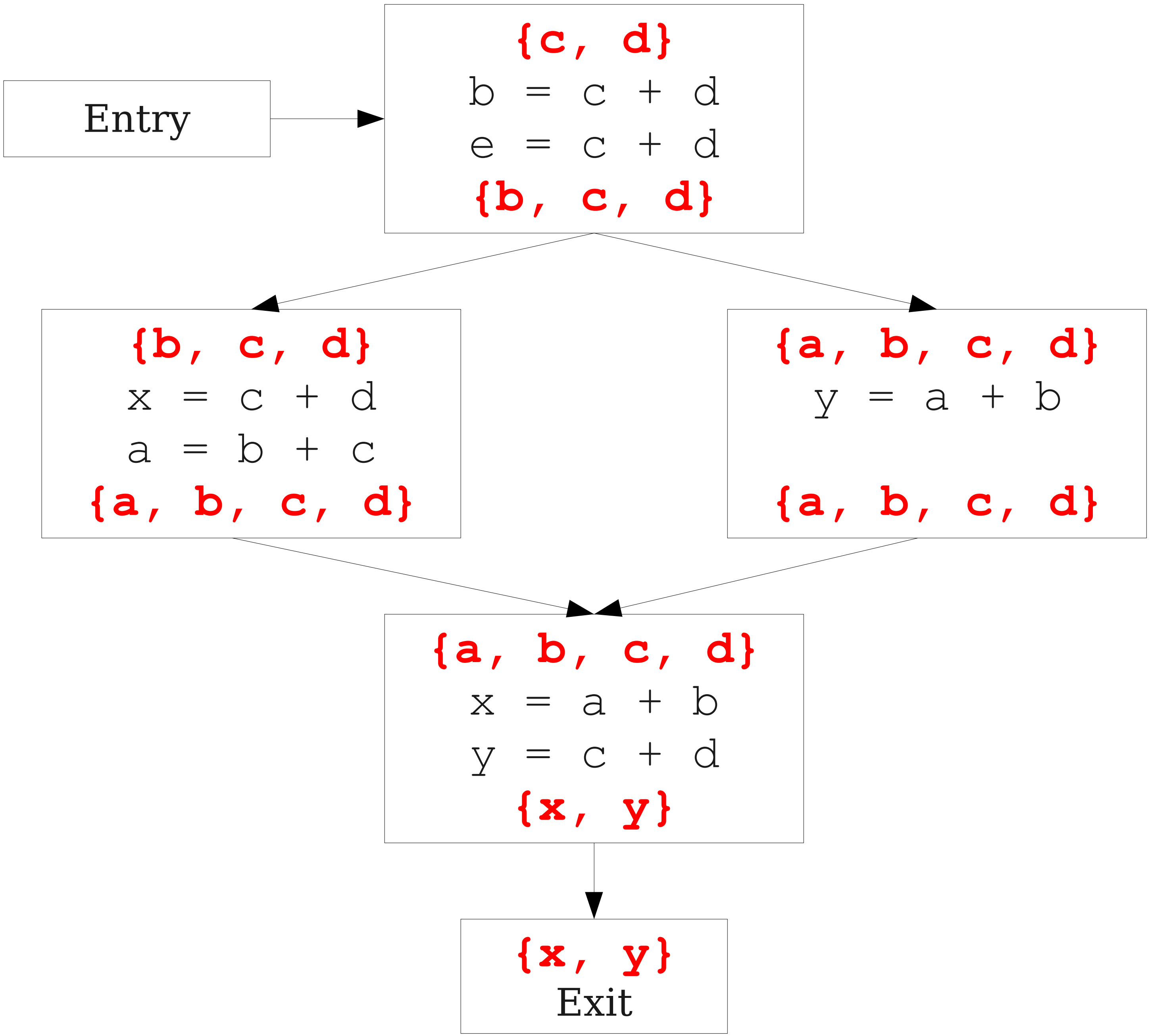

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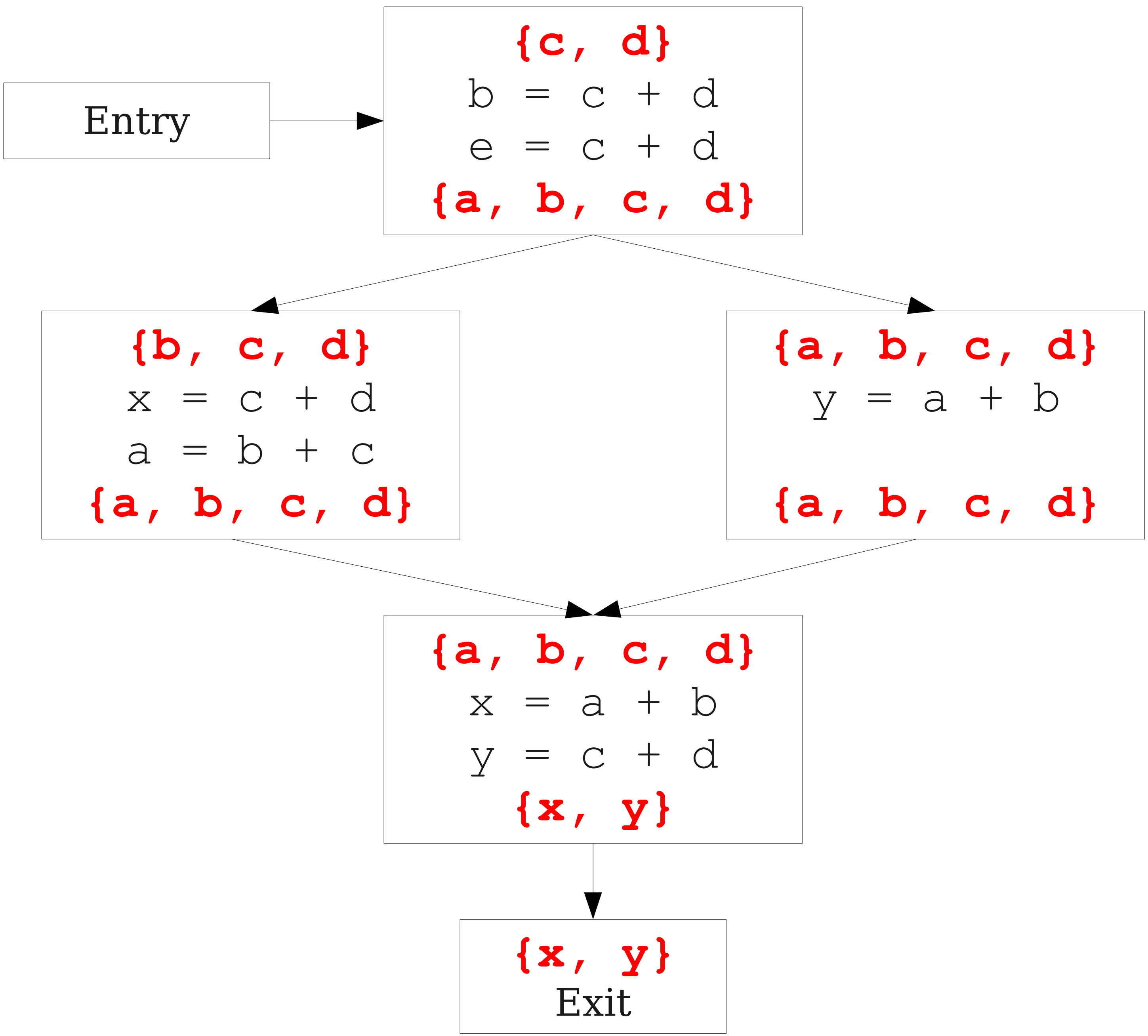

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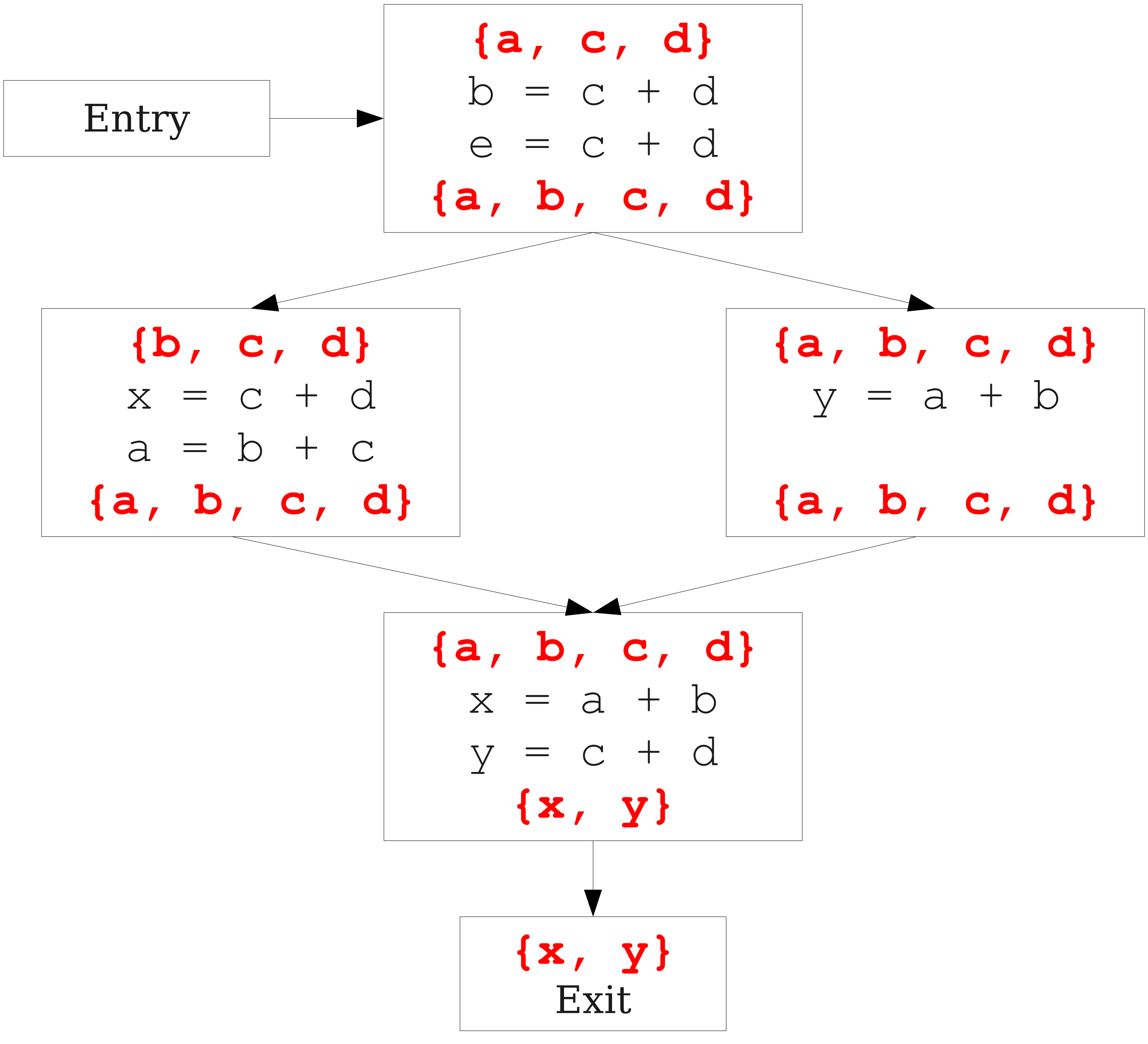

Major Changes, Part II

- In a local analysis, there is only one possible path through a basic block.
- In a global analysis, there may be **many** paths through a CFG.
- May need to recompute values multiple times as more information becomes available.
- Need to be careful when doing this not to loop infinitely!
  - (More on that later)

CFGs with Loops

- Up to this point, we've considered loop-free CFGs, which have only finitely many possible paths.
- When we add loops into the picture, this is no longer true.
- Not all possible loops in a CFG can be realized in the actual program.

CFGs with Loops

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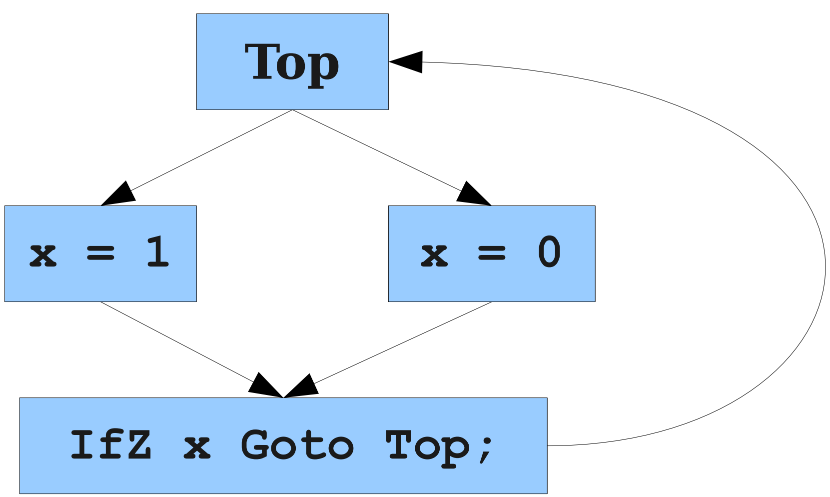
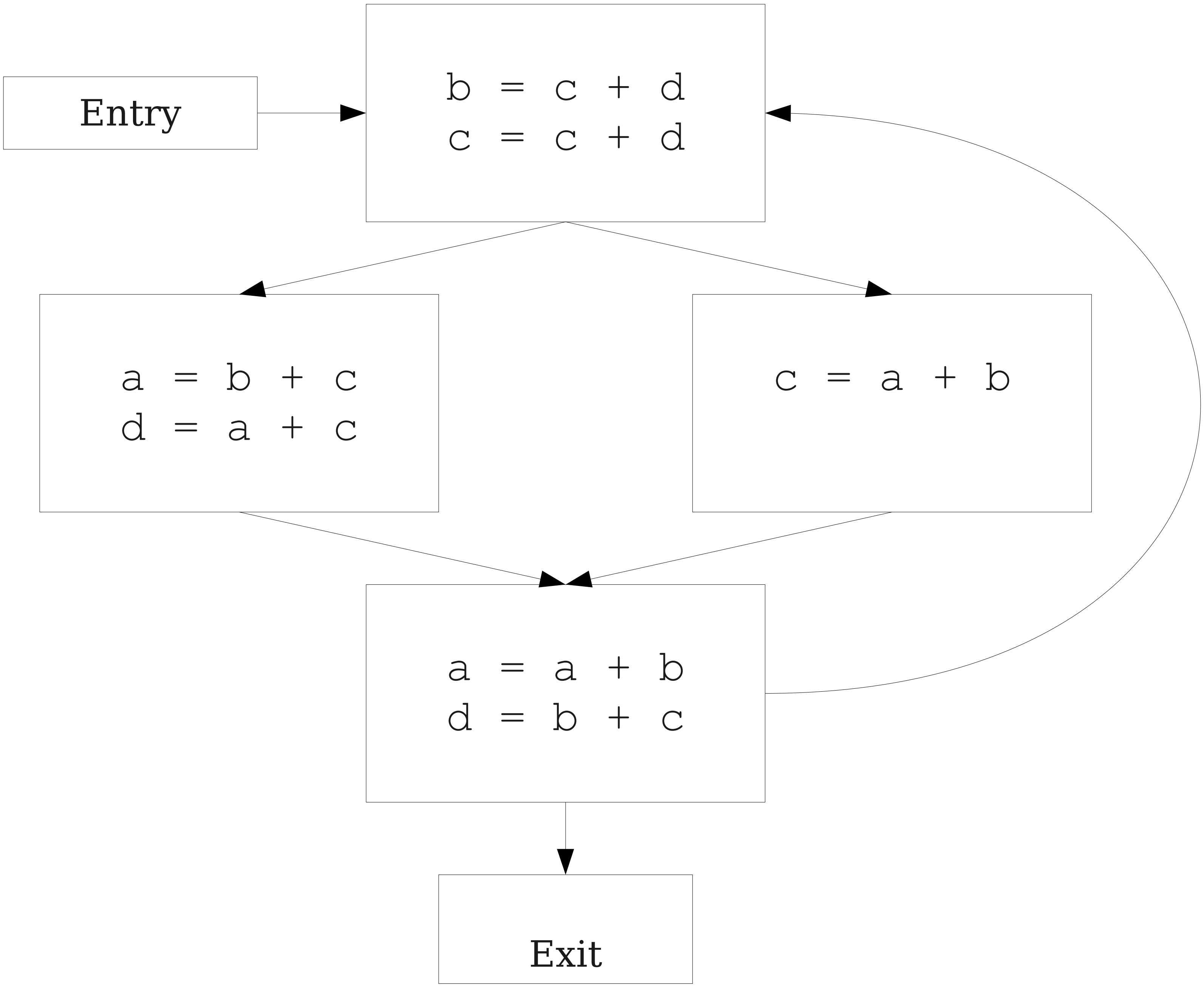


CFGs with Loops

- Up to this point, we've considered loop-free CFGs, which have only finitely many possible paths.
- When we add loops into the picture, this is no longer true.
- Not all possible loops in a CFG can be realized in the actual program.
- **Sound approximation**: Assume that every possible path through the CFG corresponds to a valid execution.
  - Includes all realizable paths, but some additional paths as well.
  - May make our analysis less precise (but still sound).
  - Makes the analysis feasible; we'll see how later.

CFGs With Loops

CFGs With Loops



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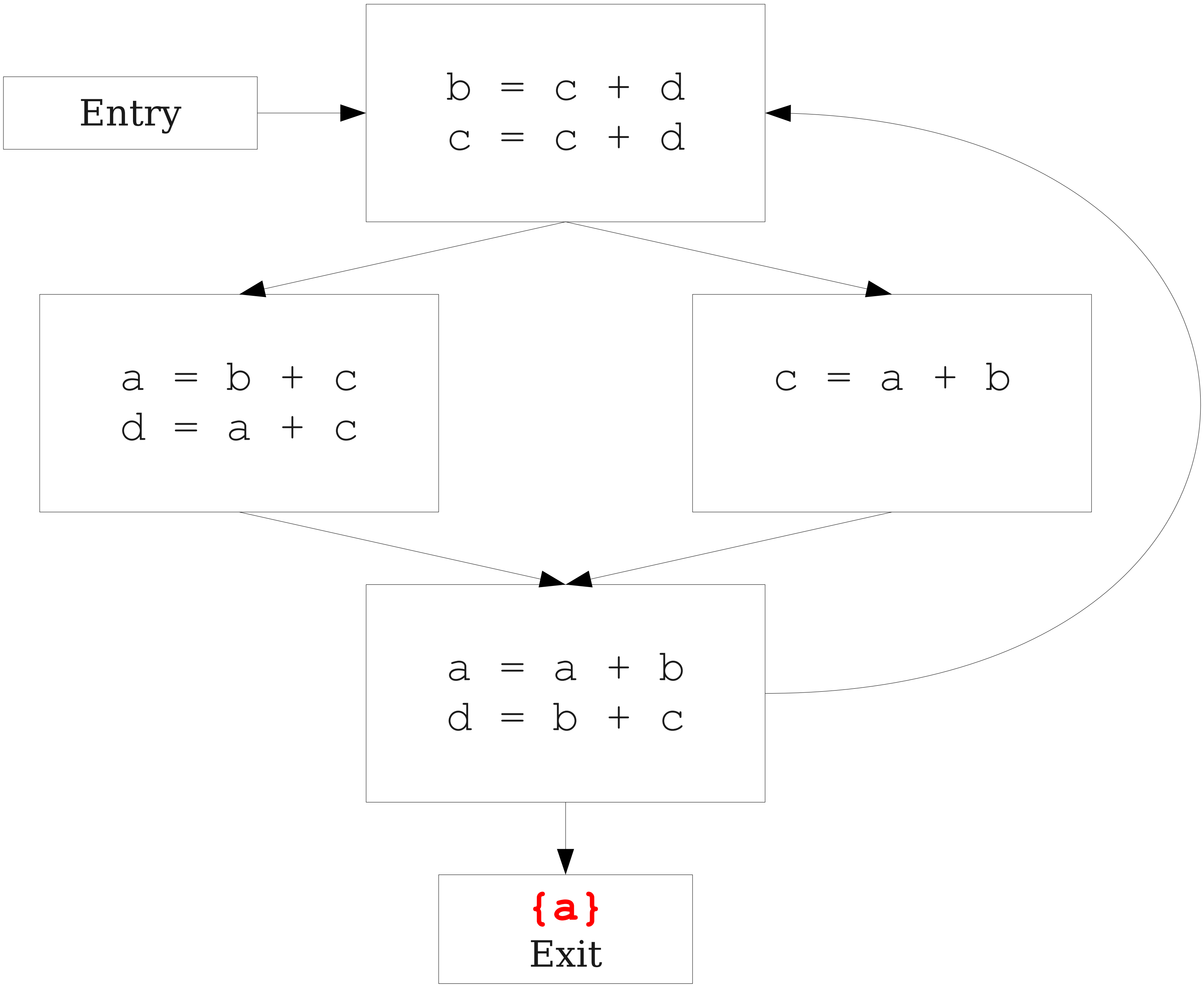
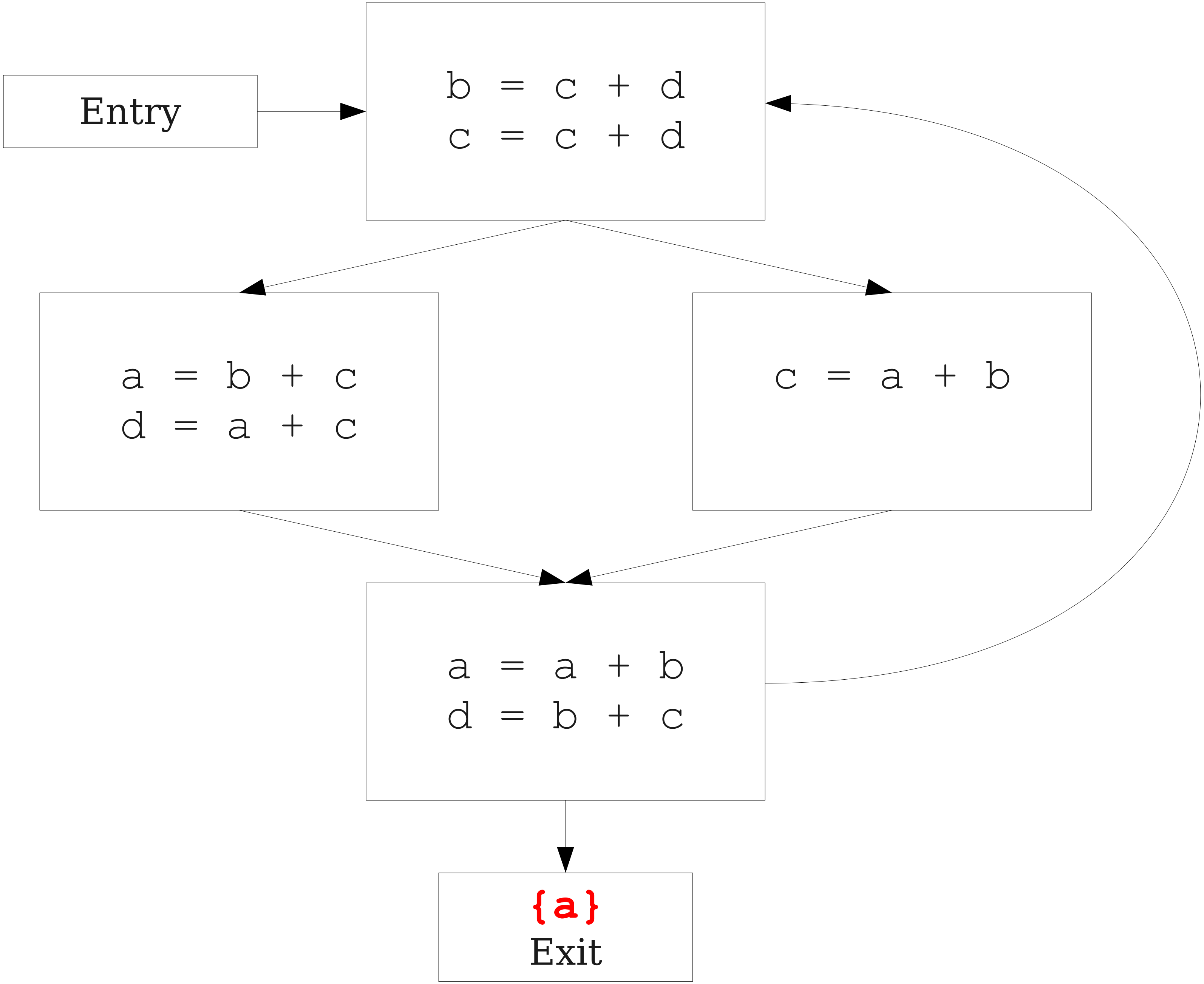


Major Changes, Part III

- In a local analysis, there is always a well-defined “first” statement to begin processing.
- In a global analysis with loops, every basic block might depend on every other basic block.
- To fix this, we need to assign initial values to all of the blocks in the CFG.

CFGs With Loops



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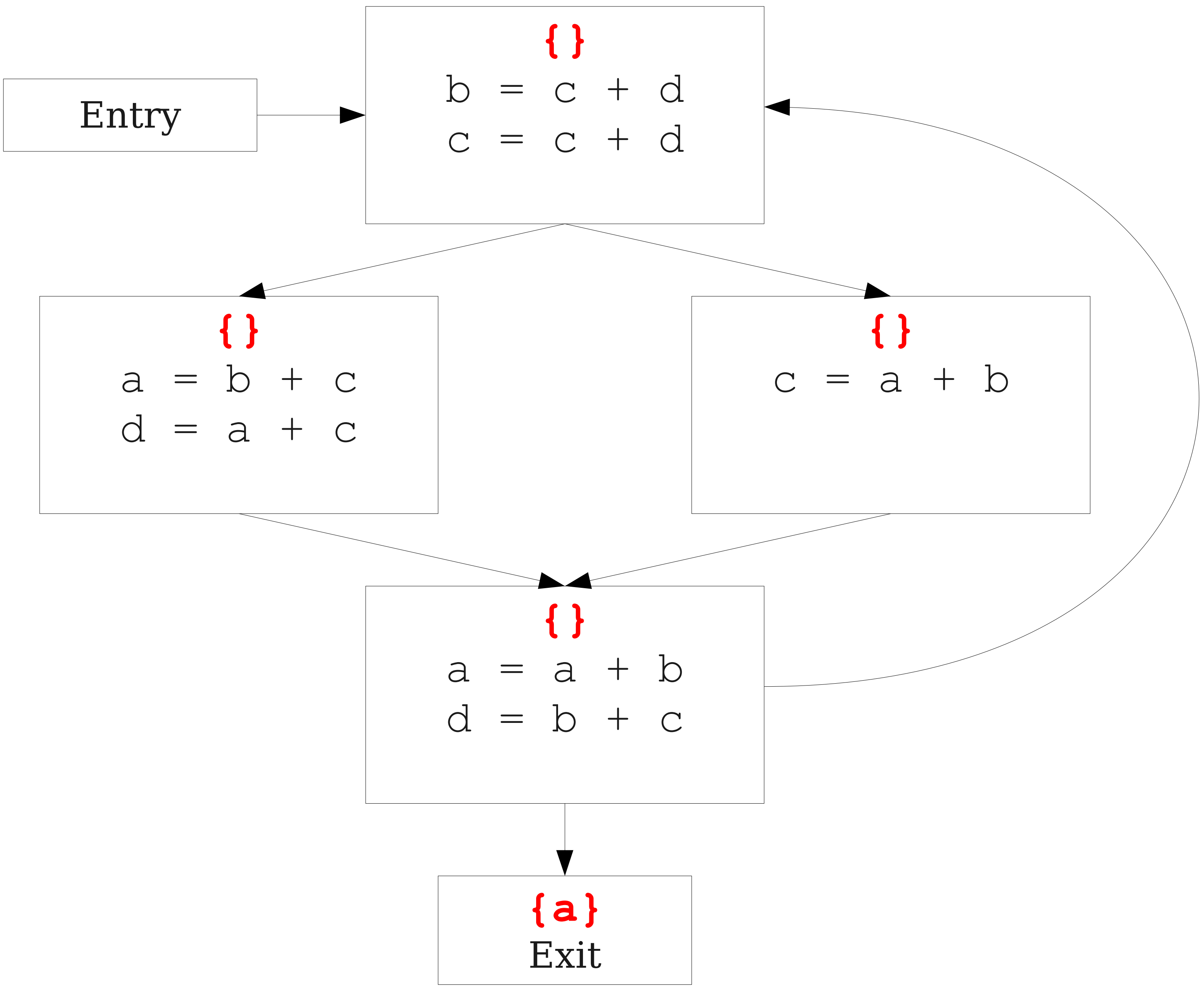

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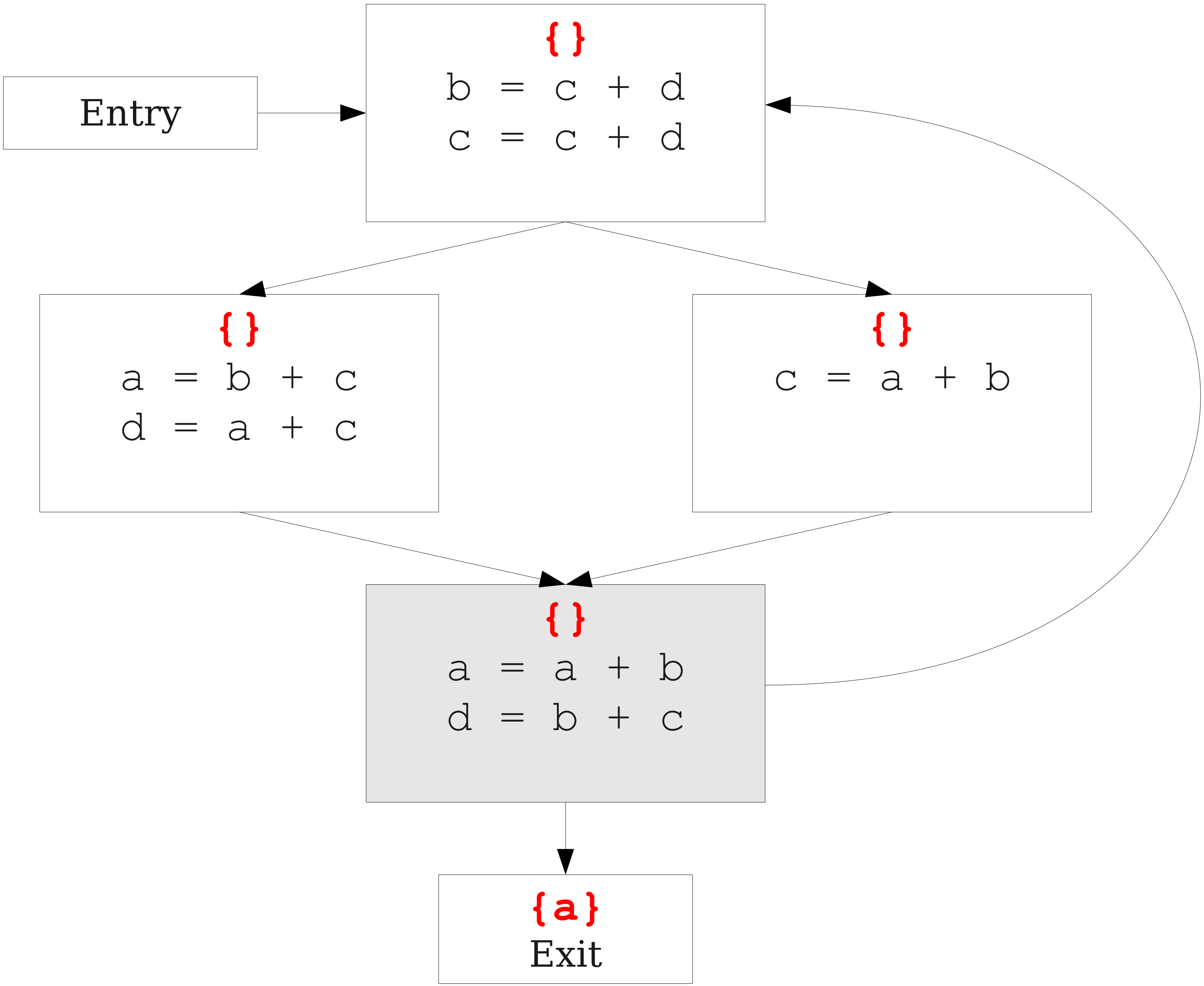

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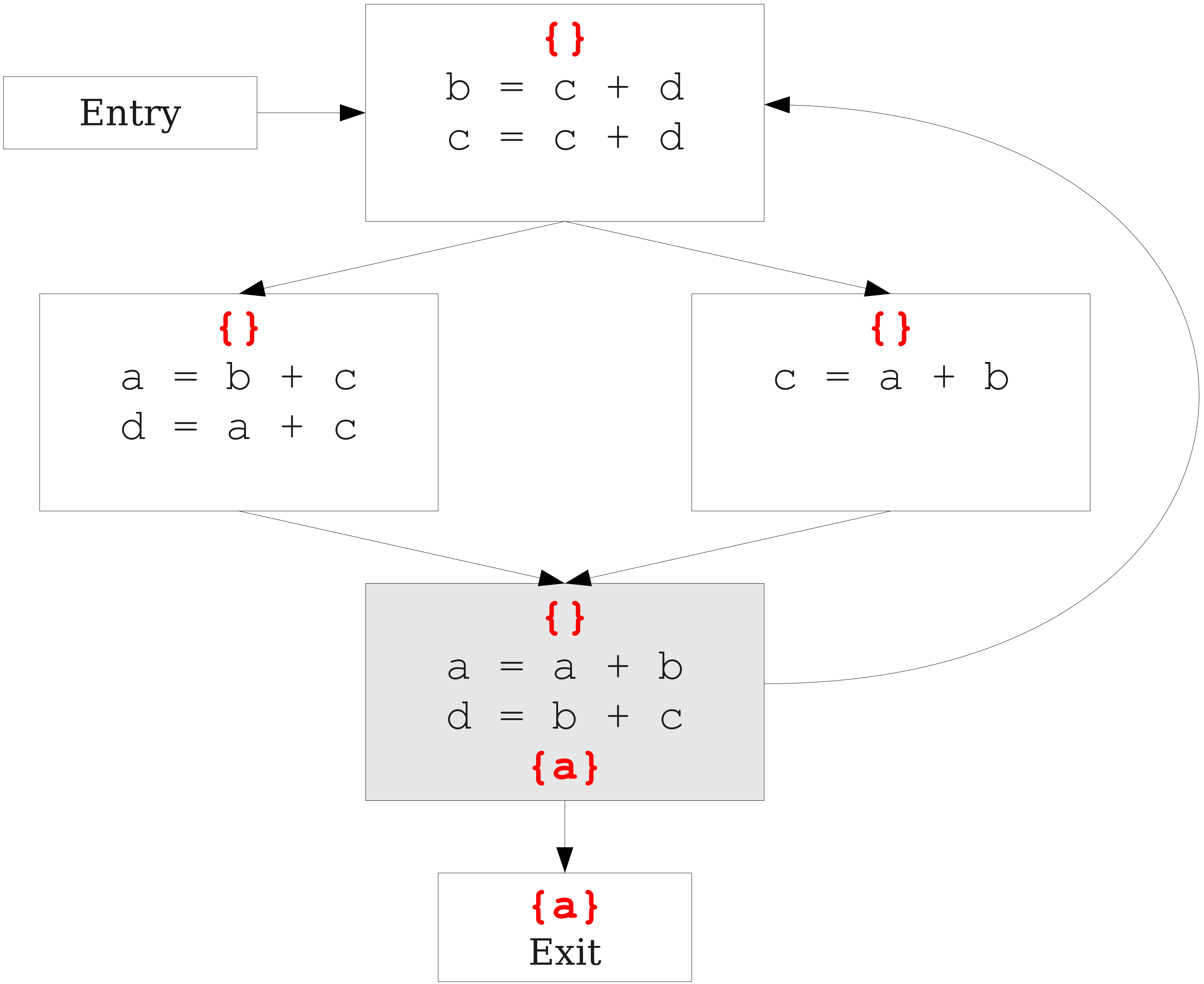

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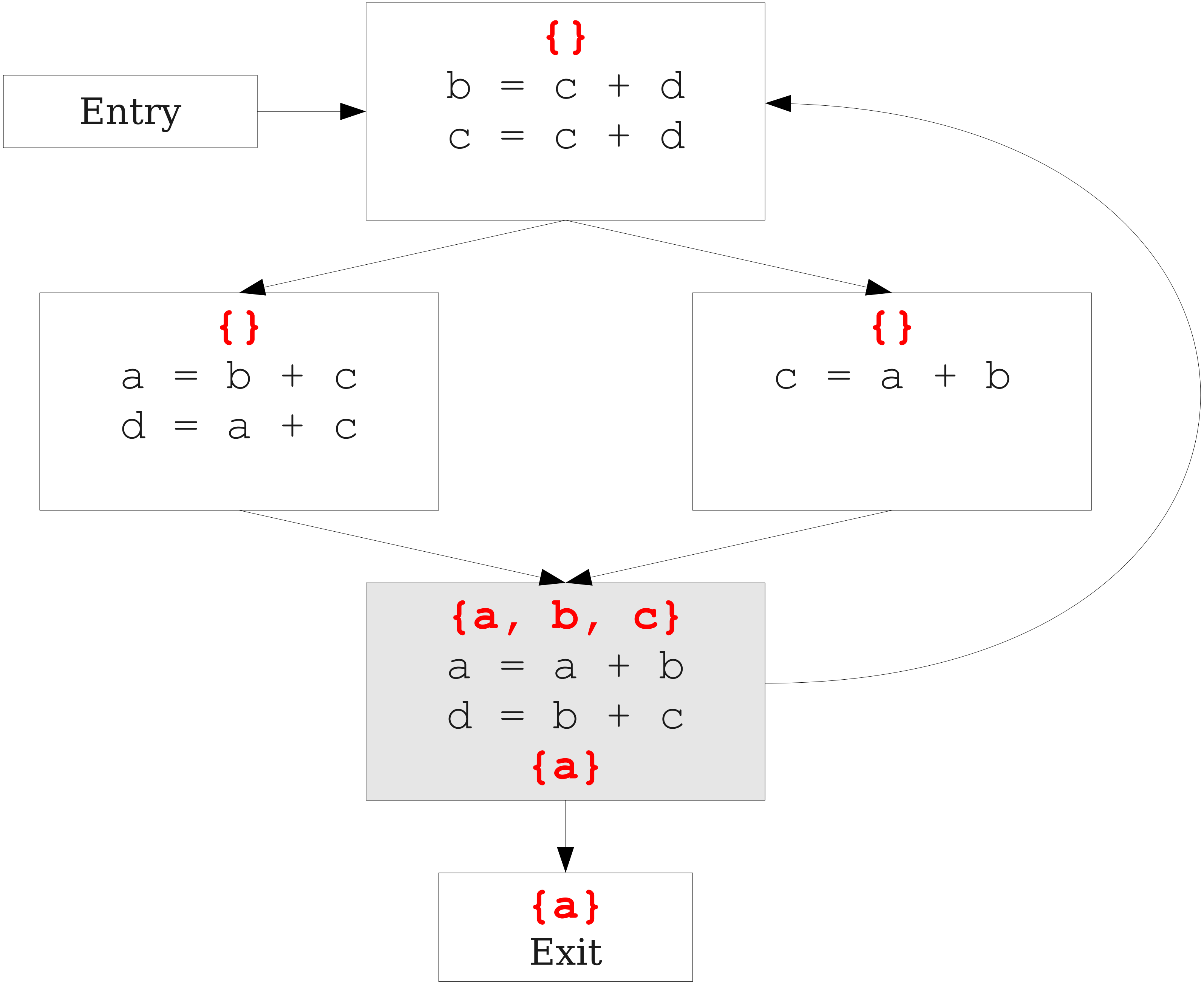

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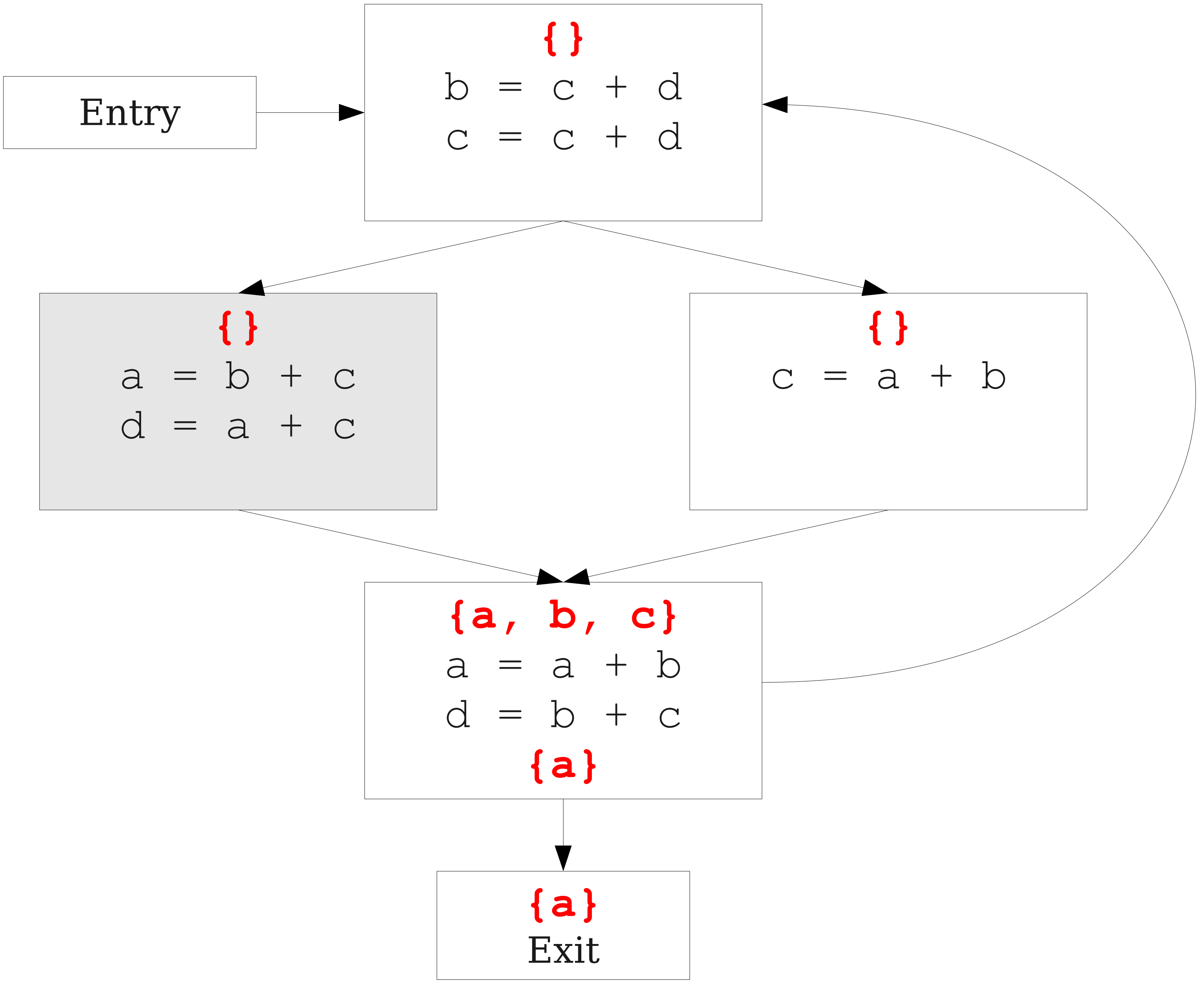

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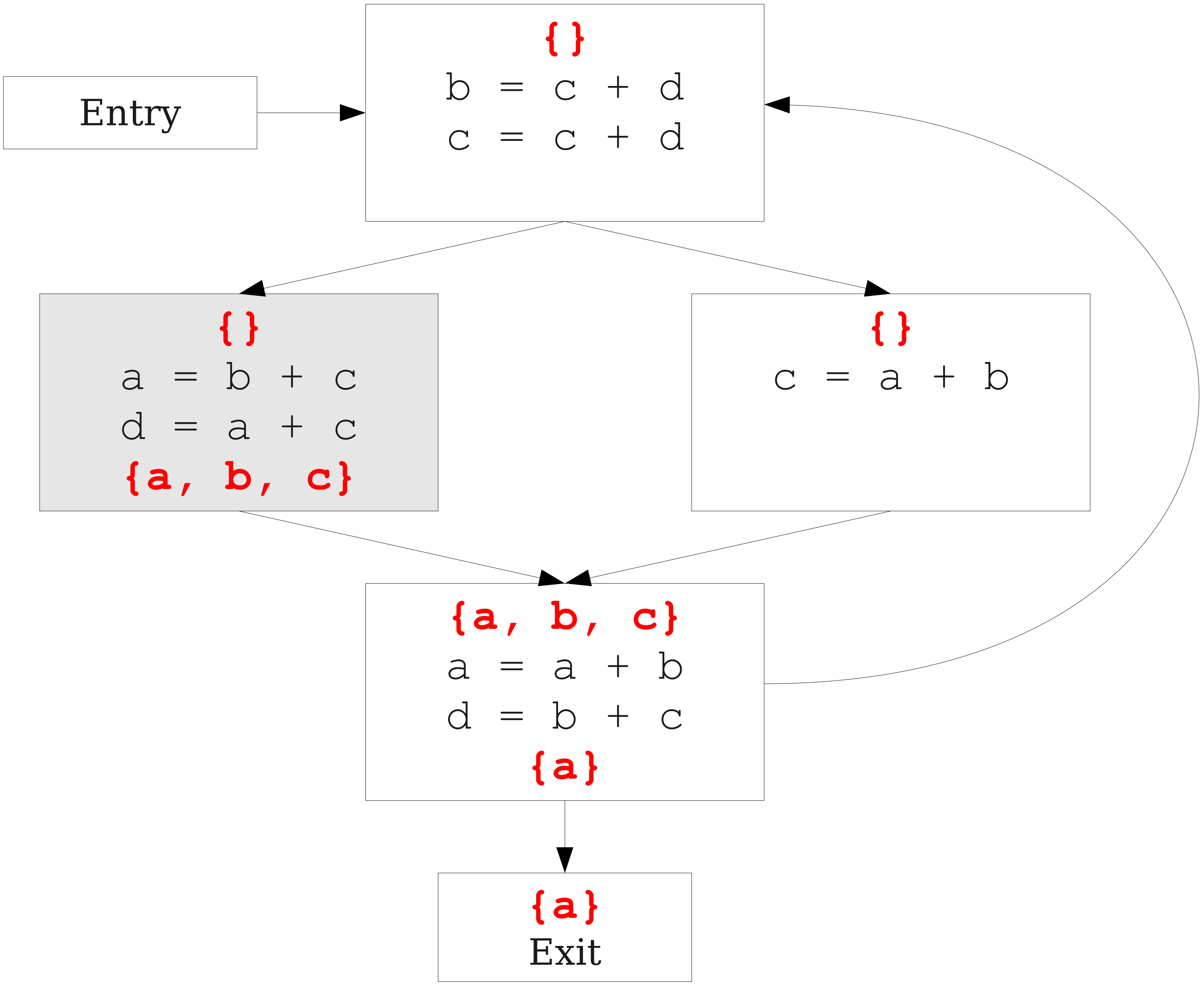

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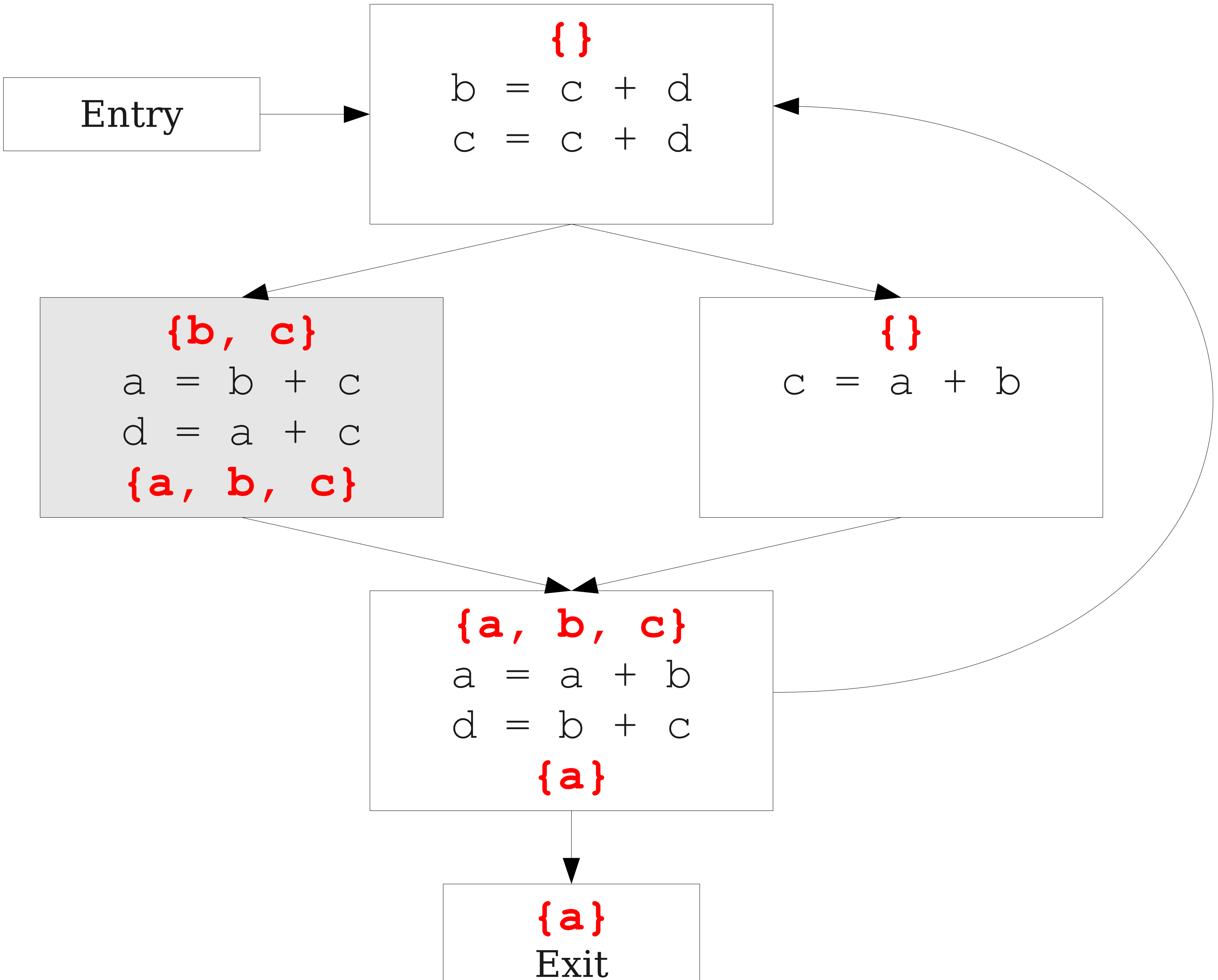

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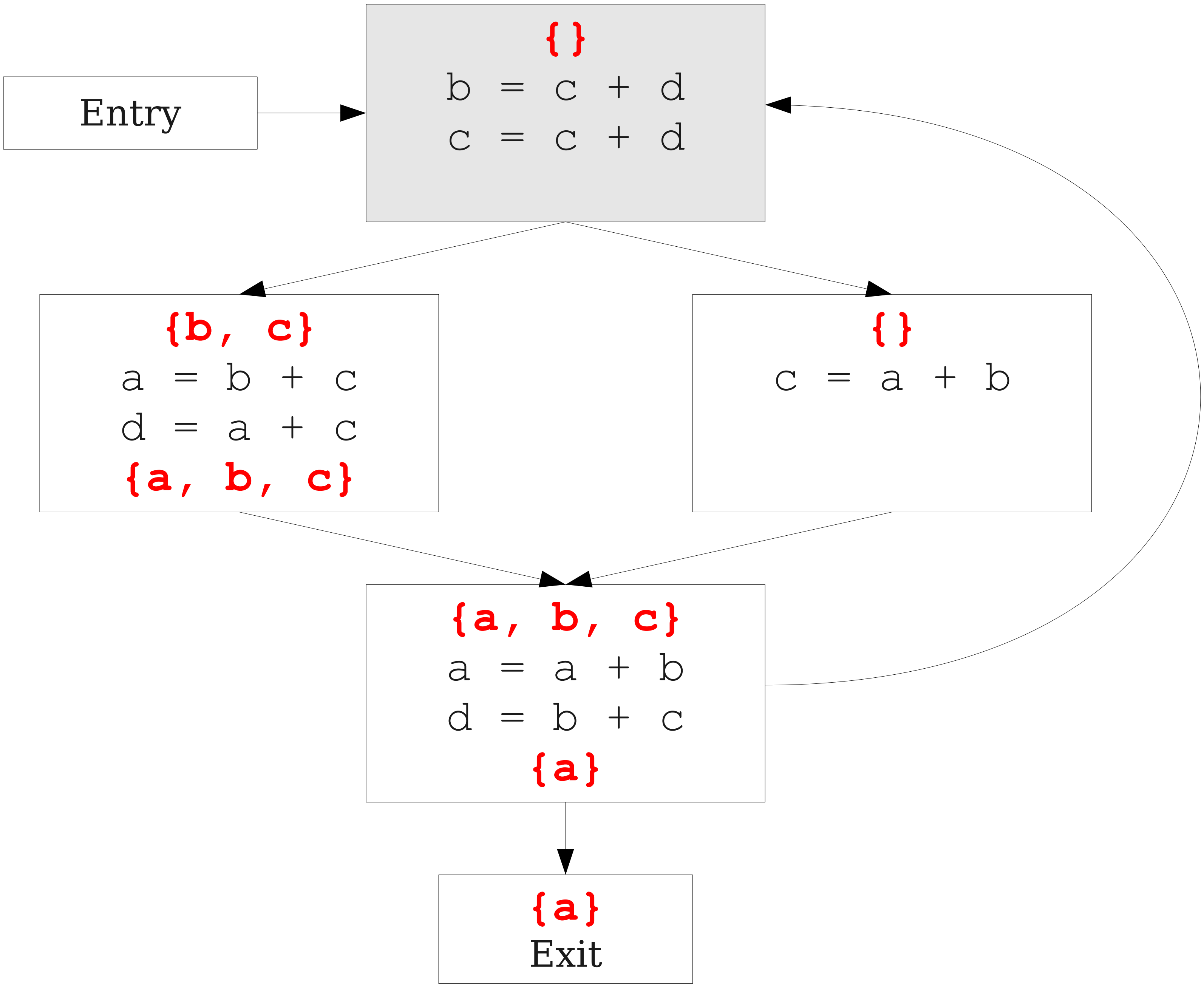

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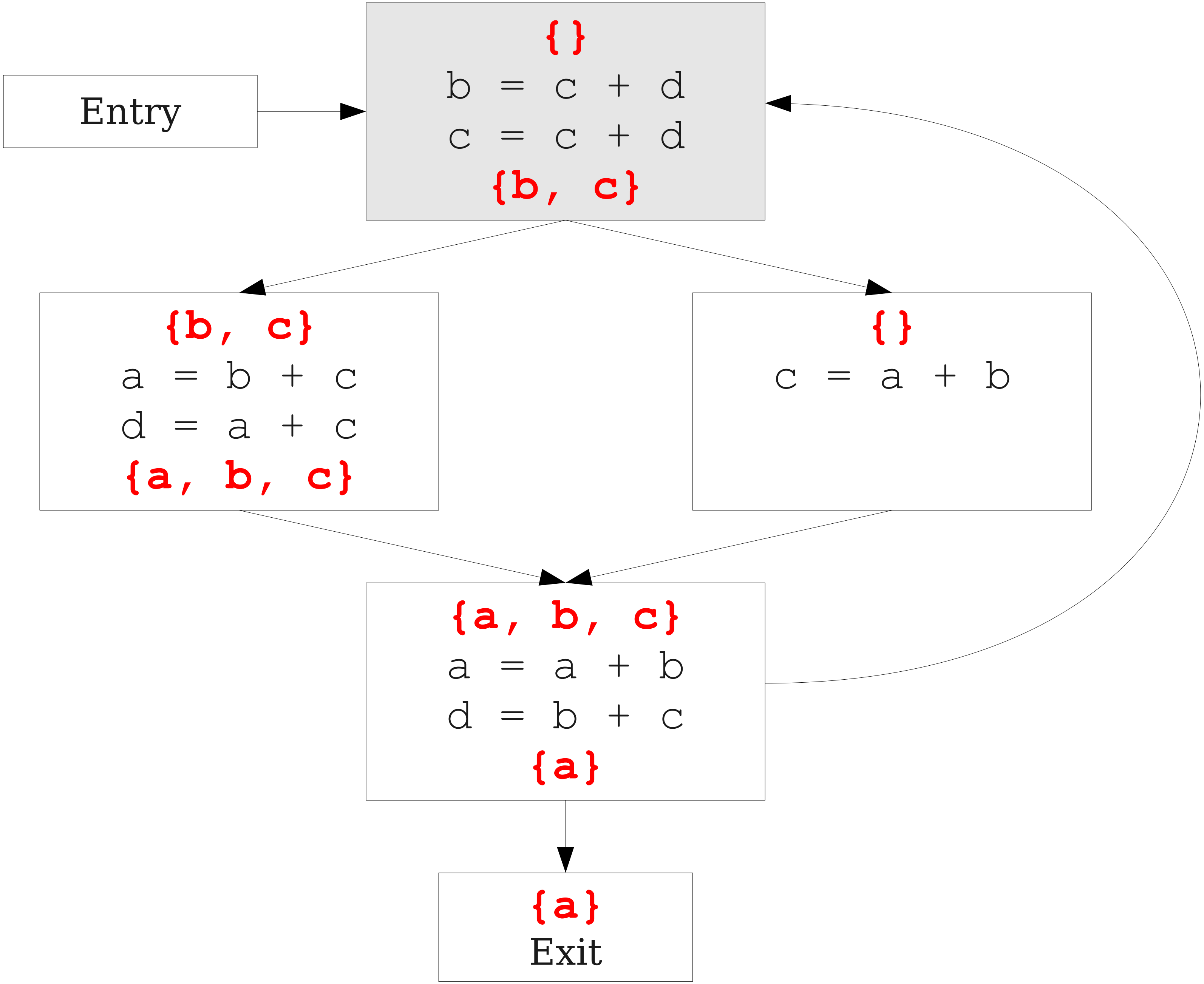

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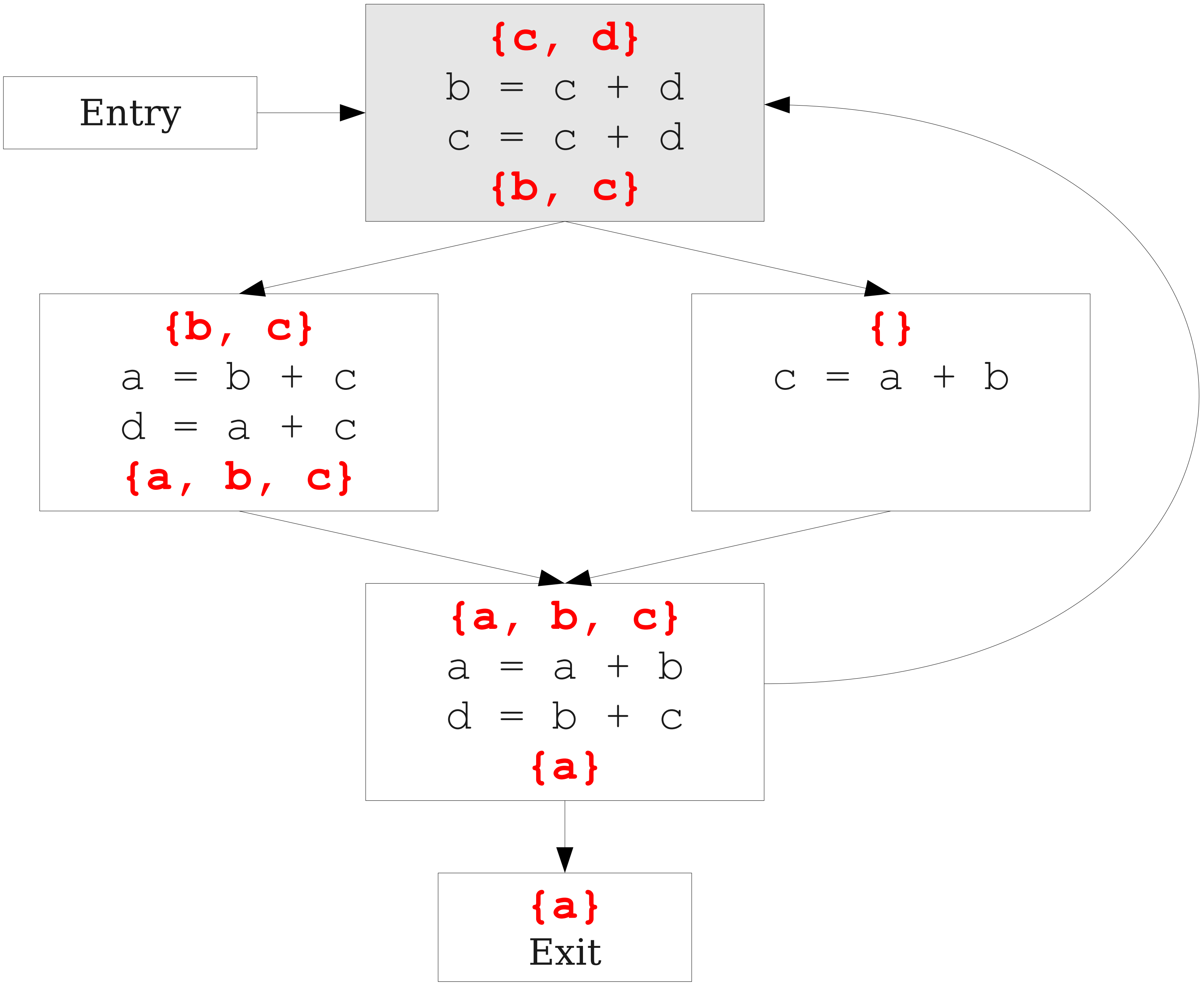

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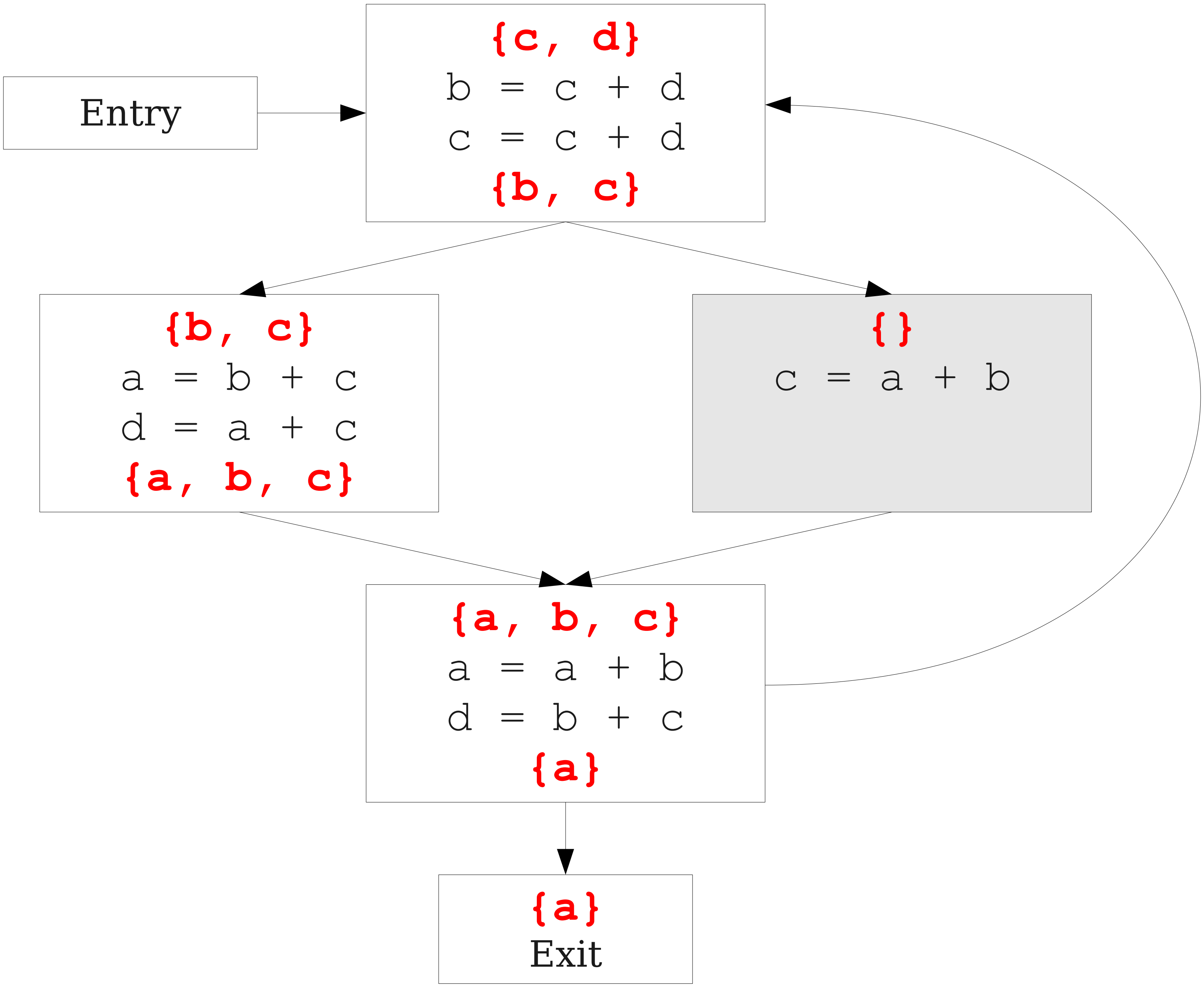

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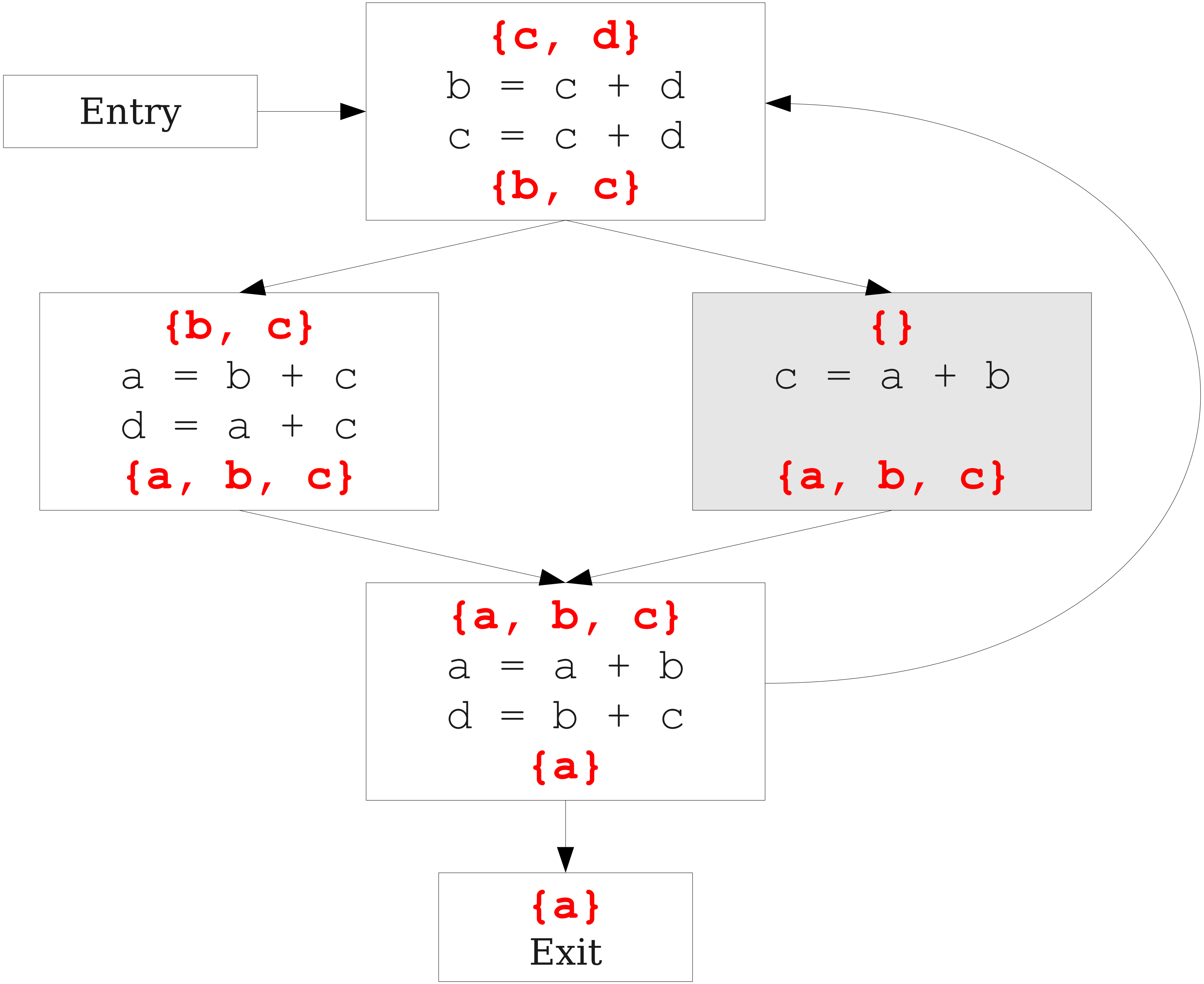

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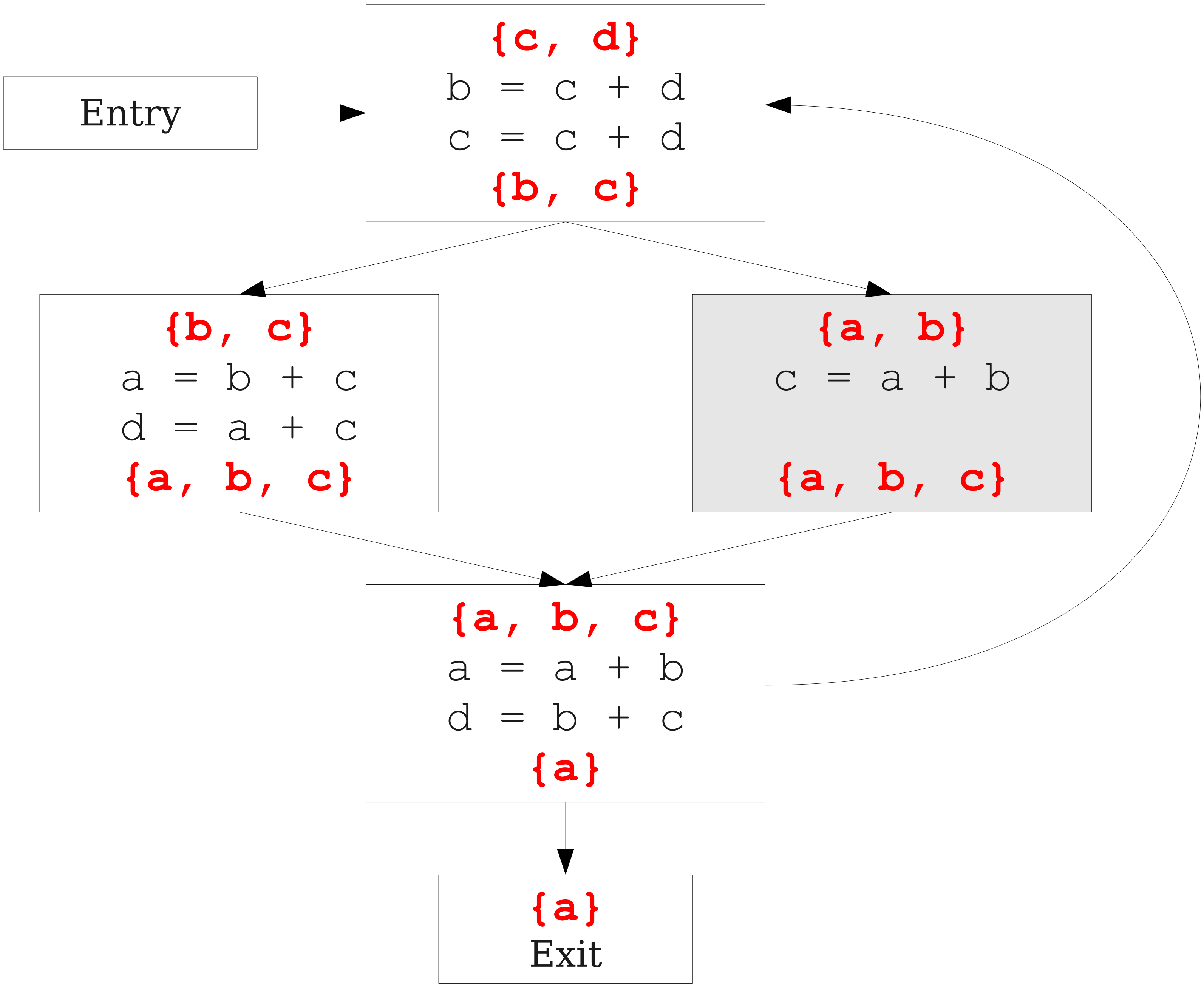

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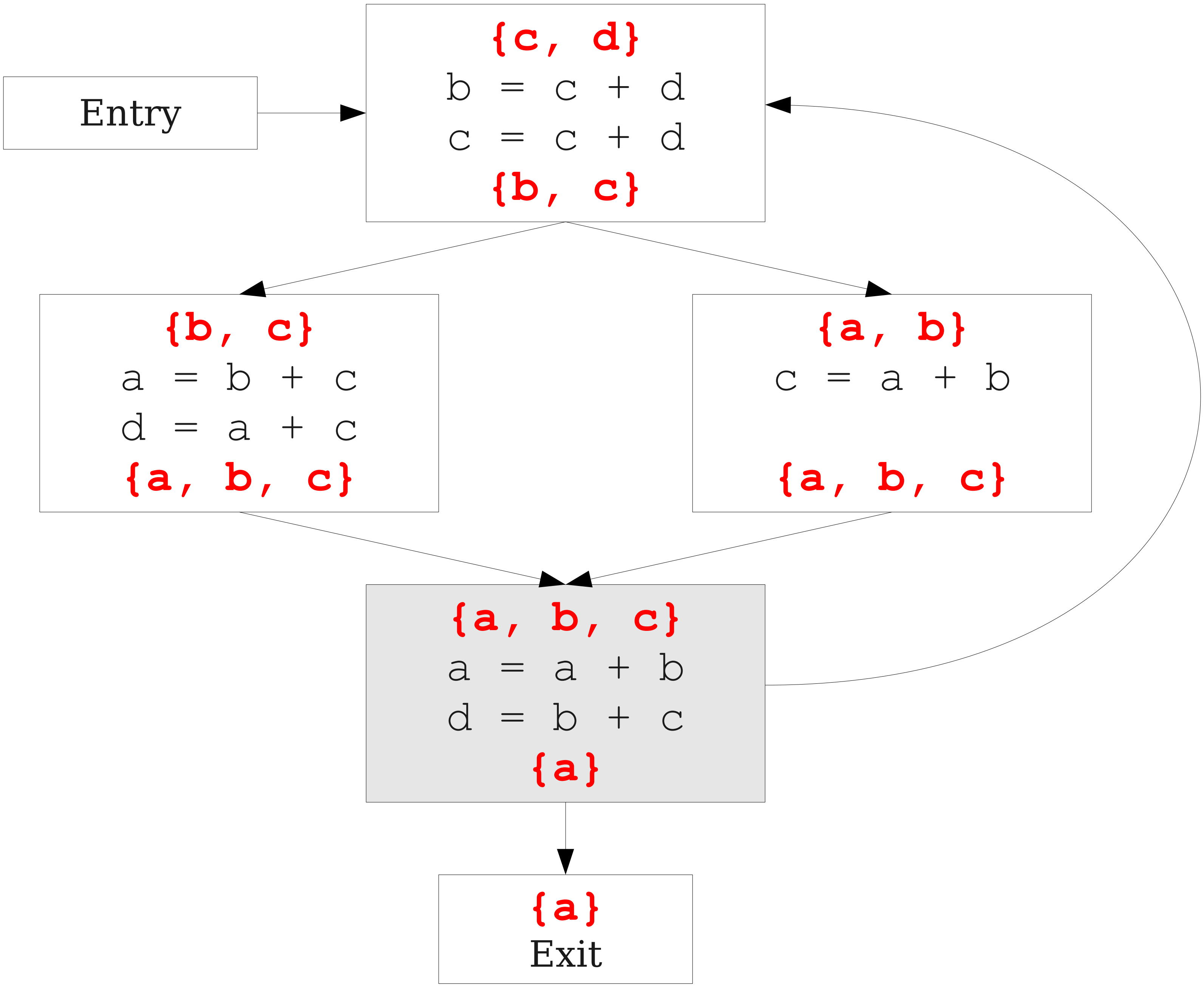

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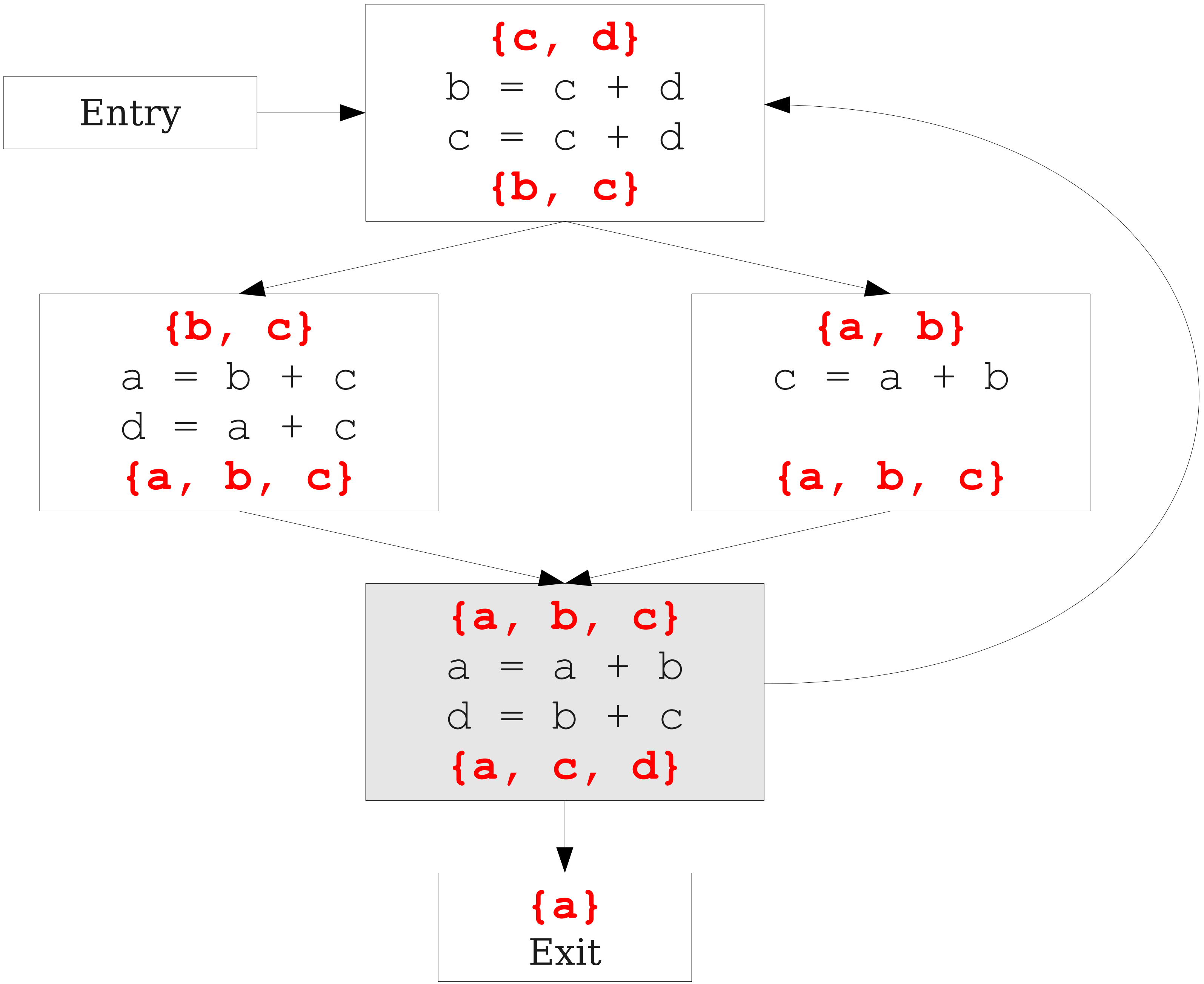

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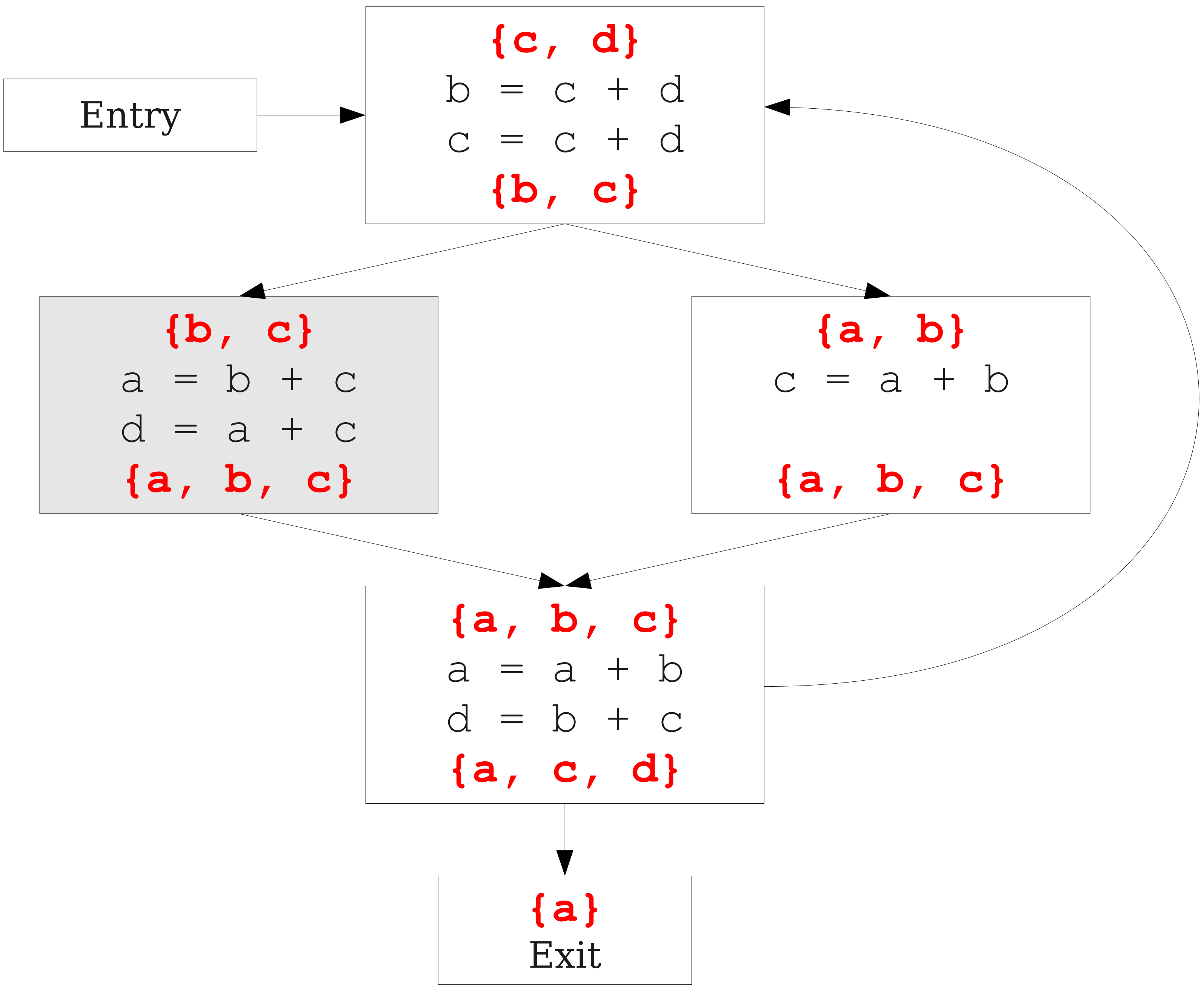

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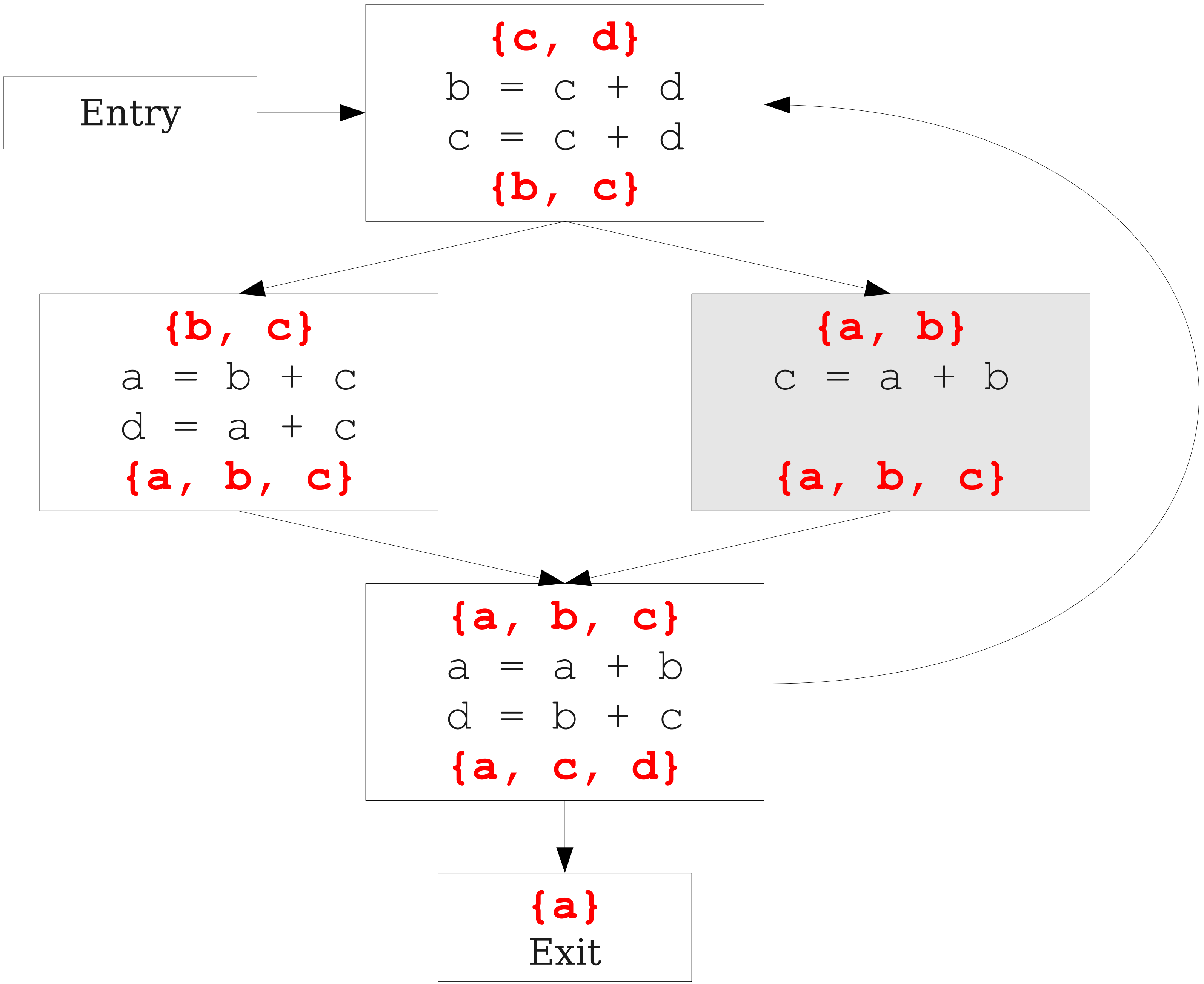

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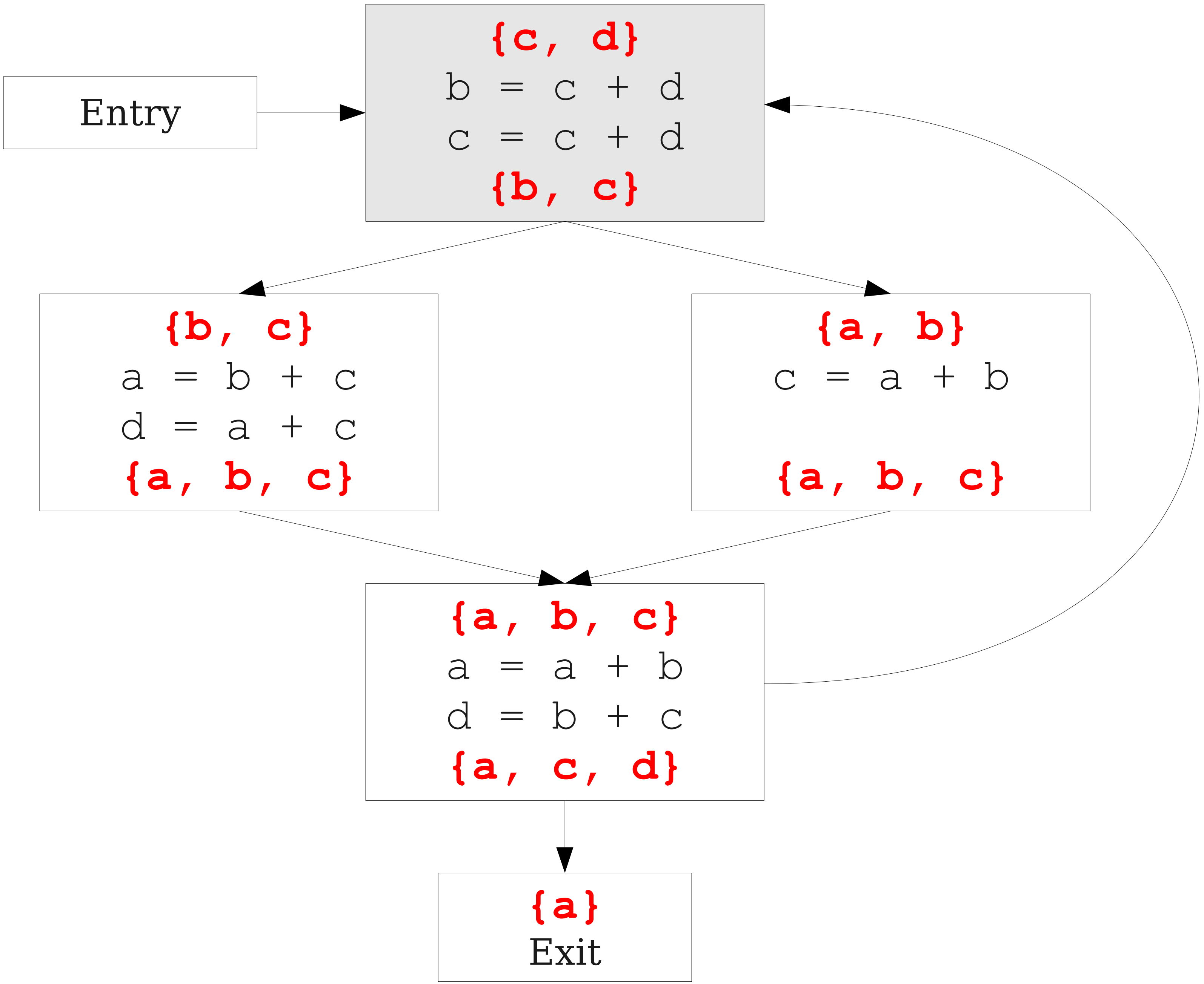

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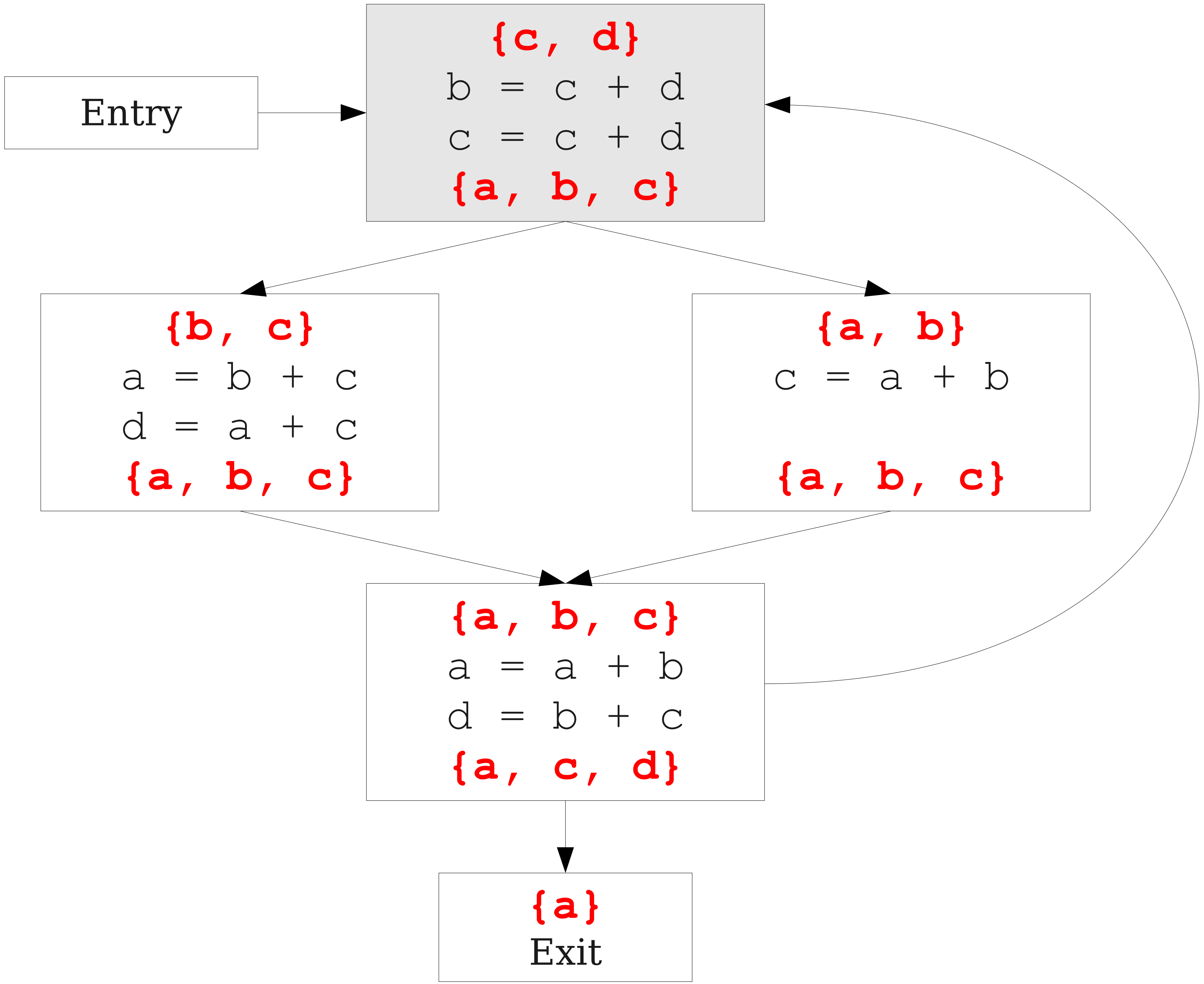

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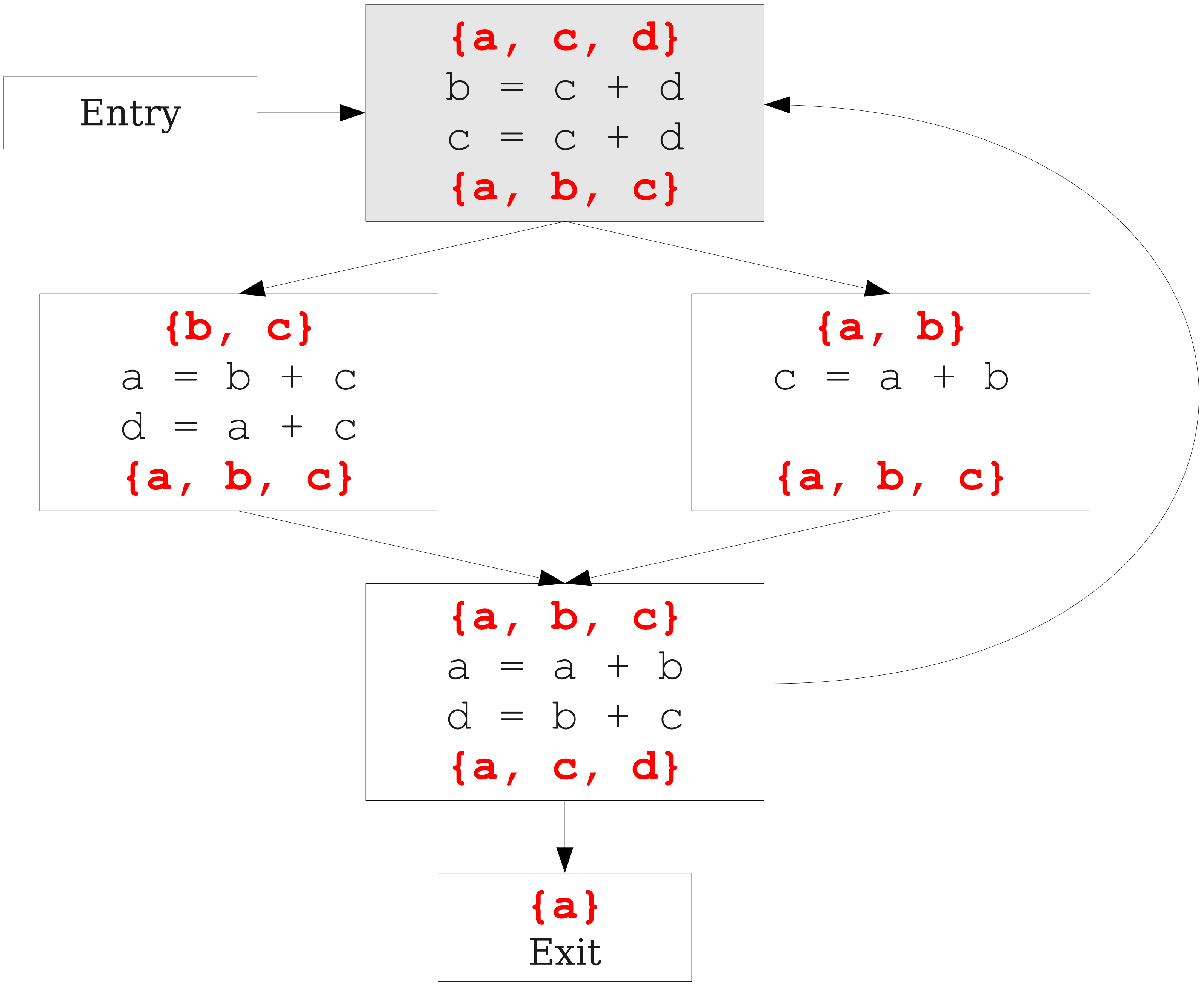

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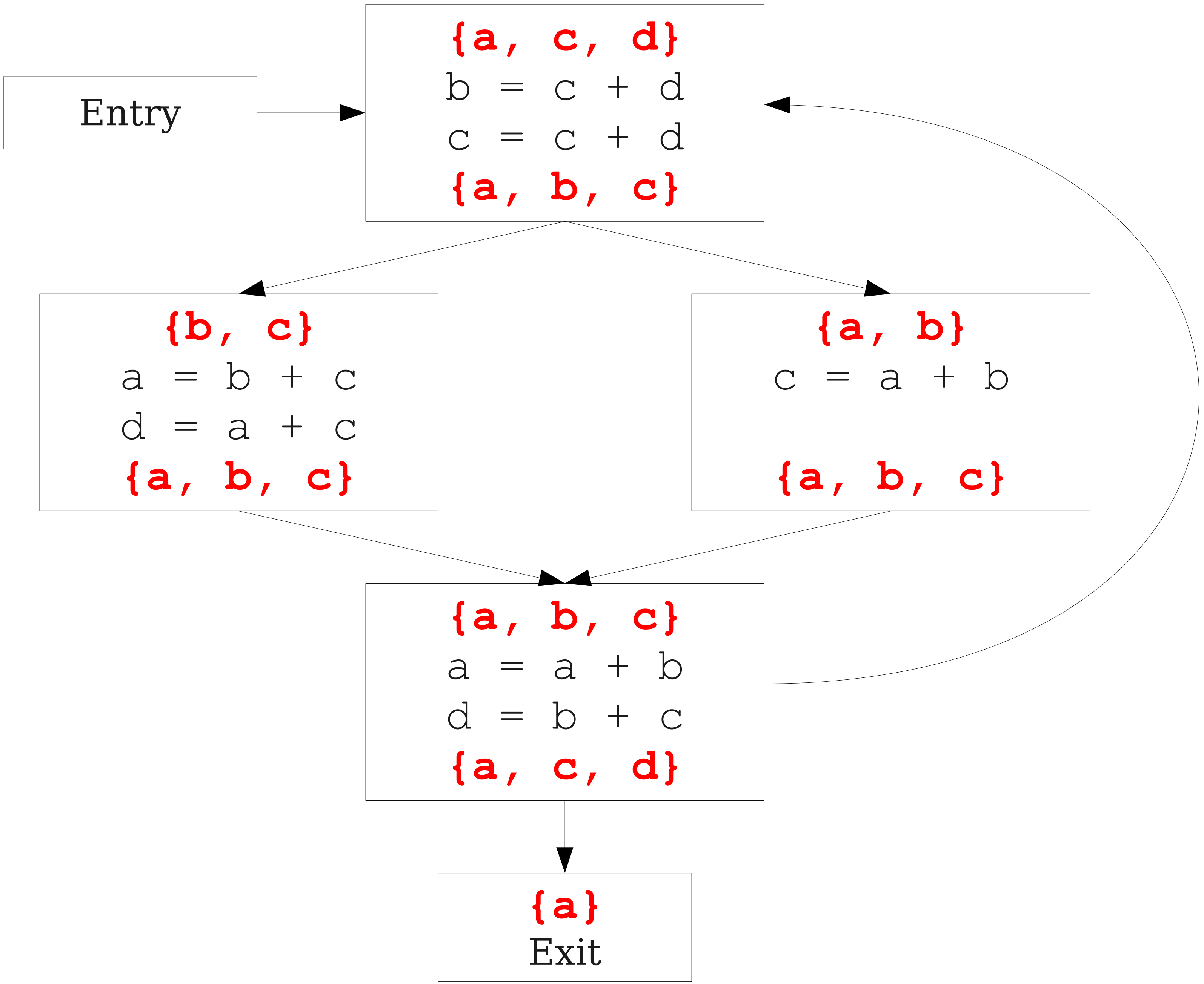


Summary of Differences

- Need to be able to handle multiple predecessors/successors for a basic block.
- Need to be able to handle multiple paths through the control-flow graph, and may need to iterate multiple times to compute the final value (but the analysis still needs to terminate!)
- Need to be able to assign each basic block a reasonable default value for before we've analyzed it.

Global Liveness Analysis

- Initially, set IN[**s**] = { } for each statement **s**.
- Set IN[**exit**] to the set of variables known to be live on exit (language-specific knowledge).
- Repeat until no changes occur:
  - For each statement **s** of the form **a = b + c**, in any order you'd like:
    - Set OUT[**s**] to set union of IN[**p**] for each successor **p** of **s**.
    - Set IN[**s**] to (OUT[**s**] – **a**) ∪ {**b**, **c**}.
- **Yet another fixed-point iteration!**

Why Does This Work?

- To show correctness, we need to show that
  - the algorithm eventually terminates, and
  - when it terminates, it has a sound answer.
- Termination argument:
  - Once a variable is discovered to be live during some point of the analysis, it always stays live.
  - Only finitely many variables and finitely many places where a variable can become live.
- Soundness argument (sketch):
  - Each individual rule, applied to some set, correctly updates liveness in that set.
  - When computing the union of the set of live variables, a variable is only live if it was live on some path leaving the statement.

Theory to the Rescue

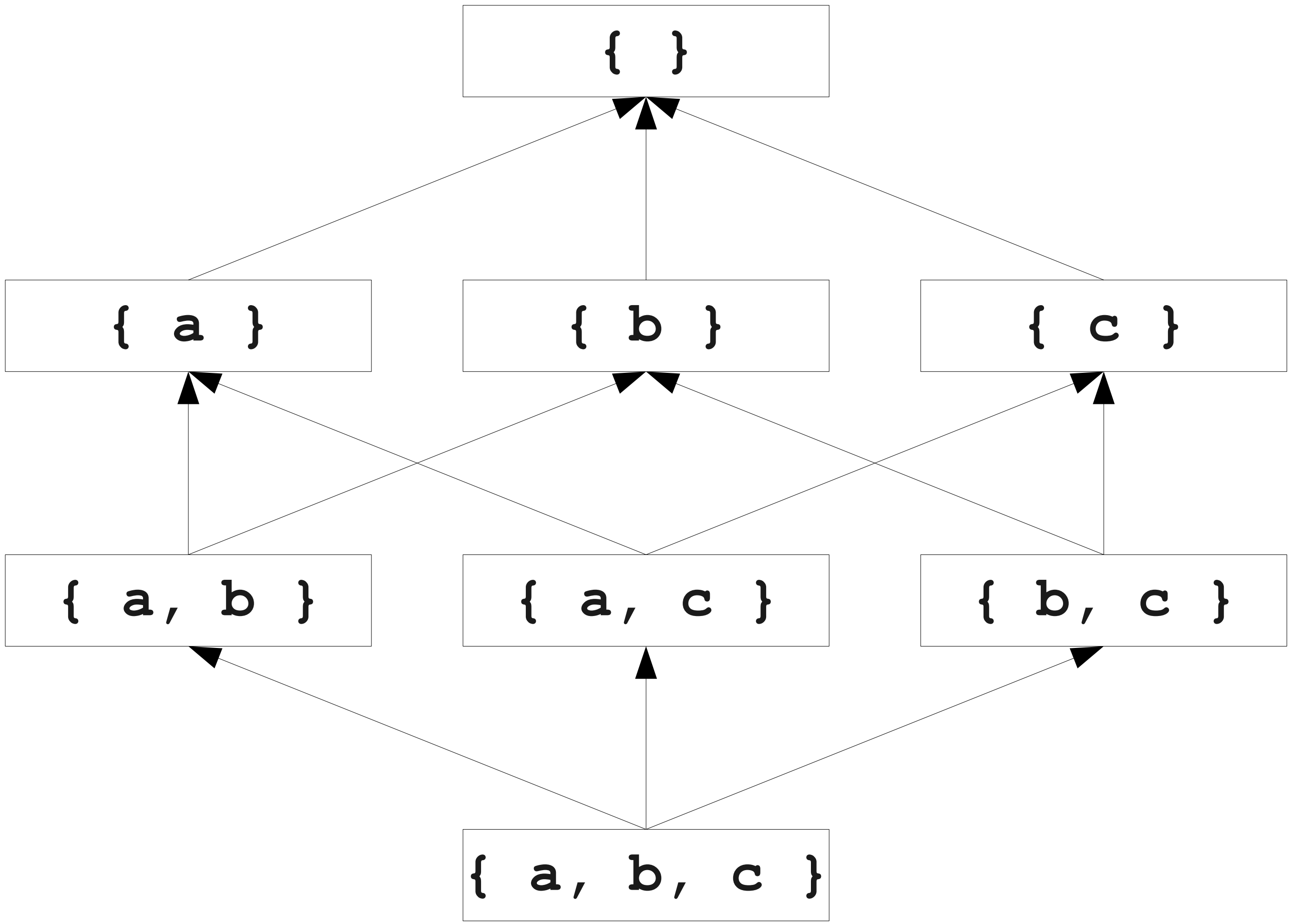
- Building up all of the machinery to design this analysis was tricky.
- The key ideas, however, are mostly independent of the analysis:
  - We need to be able to compute functions describing the behavior of each statement.
  - We need to be able to merge several subcomputations together.
  - We need an initial value for all of the basic blocks.
- There is a beautiful formalism that captures many of these properties.

Meet Semilattices

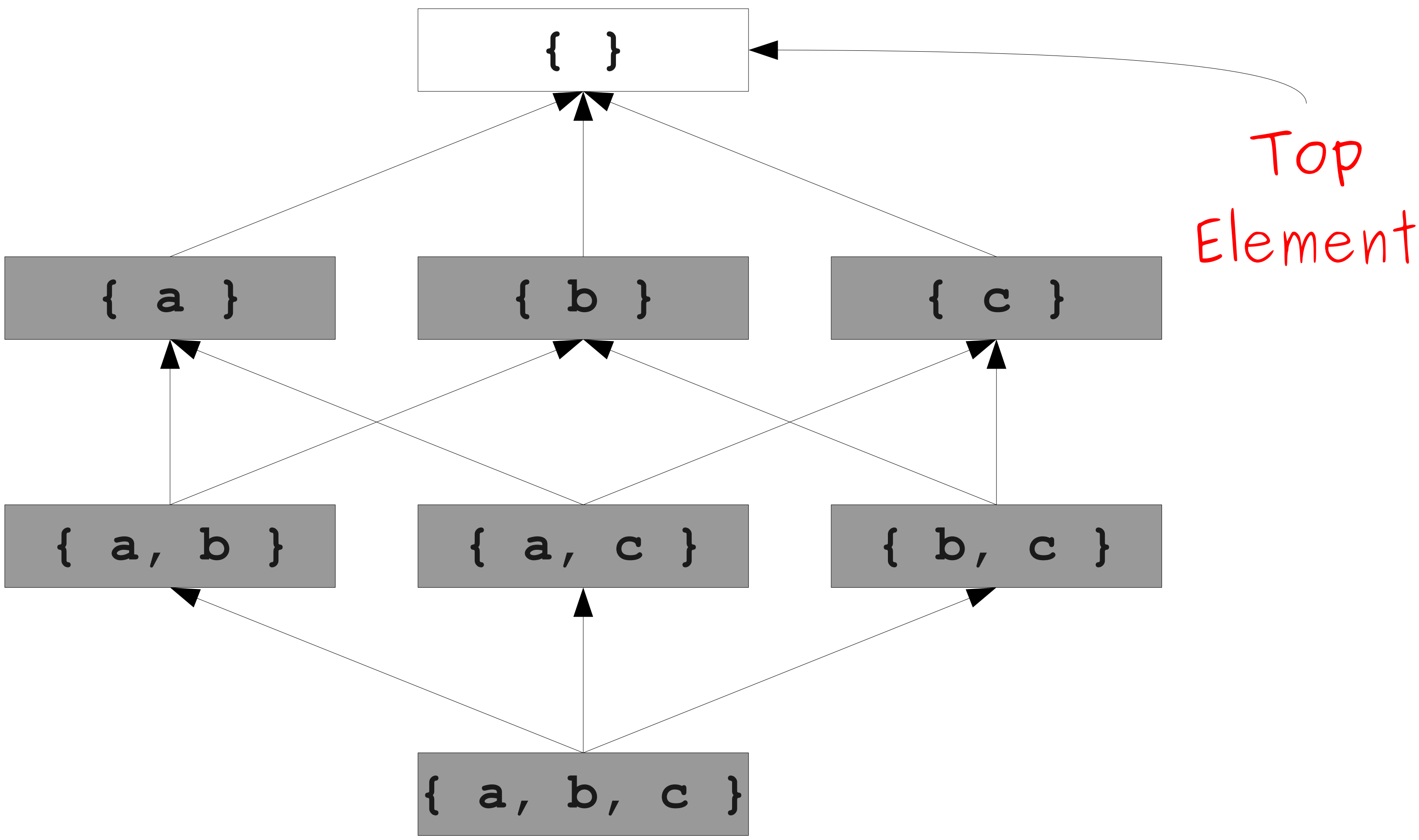
- A **meet semilattice** is a ordering defined on a set of elements.
- Any two elements have some **meet** that is the largest element smaller than both elements.
- There is a unique **top element**, which is larger than all other elements.
- Intuitively:
  - The meet of two elements represents combining information from two elements.
  - The top element element represents “no information yet” or “the least conservative possible answer.”

Meet Semilattices for Liveness

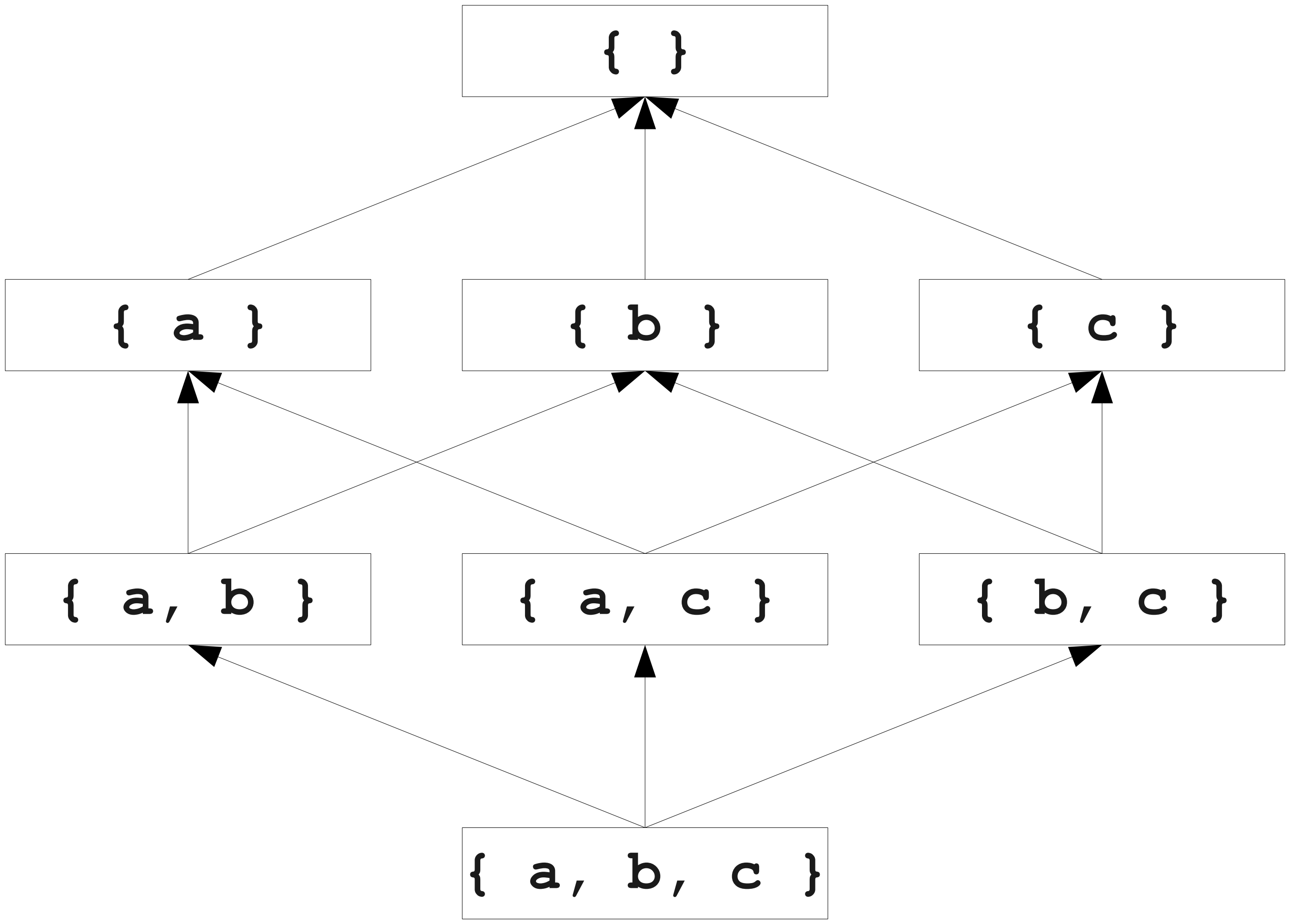
Meet Semilattices for Liveness



Meet Semilattices for Liveness



Meet Semilattices for Liveness



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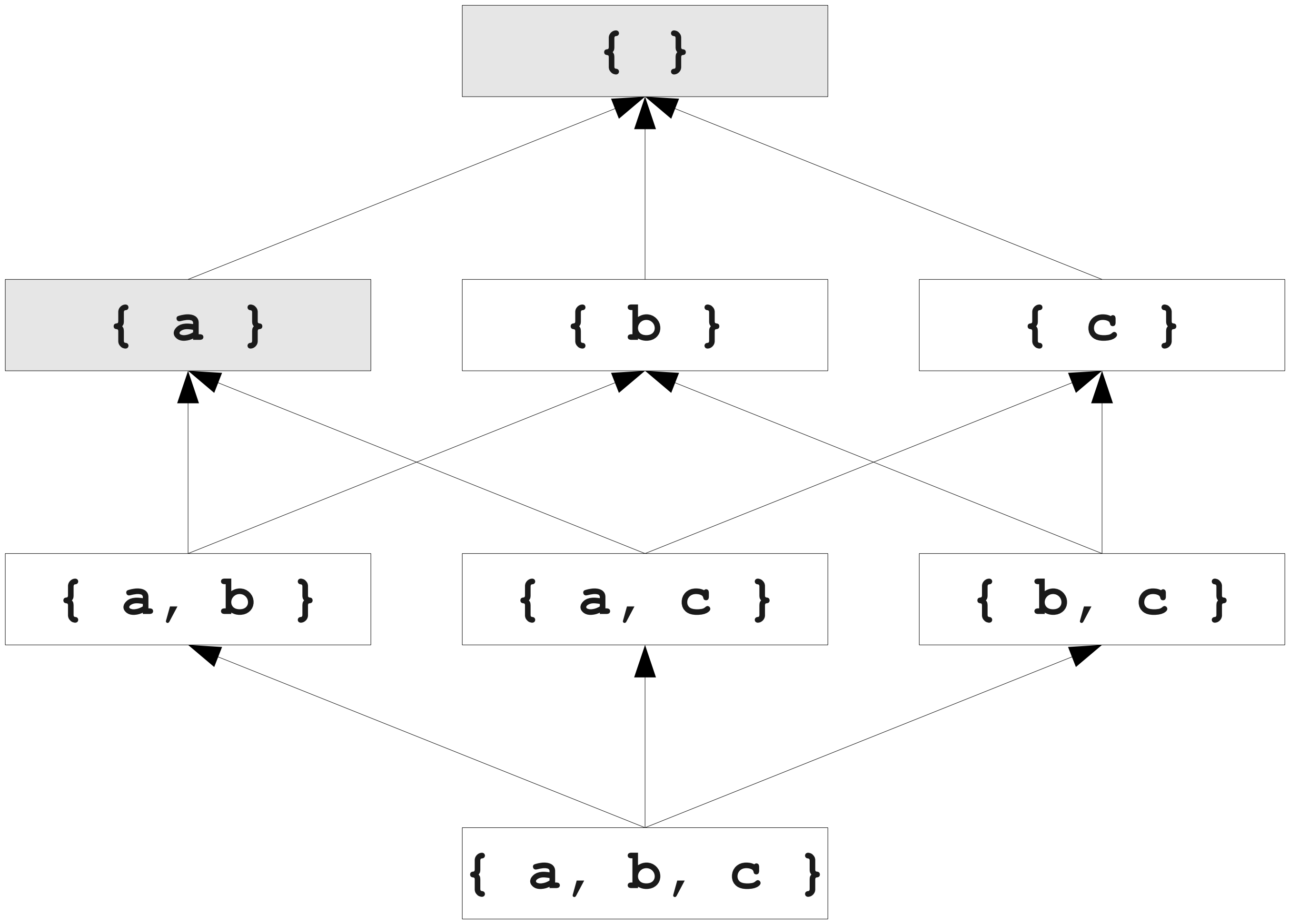

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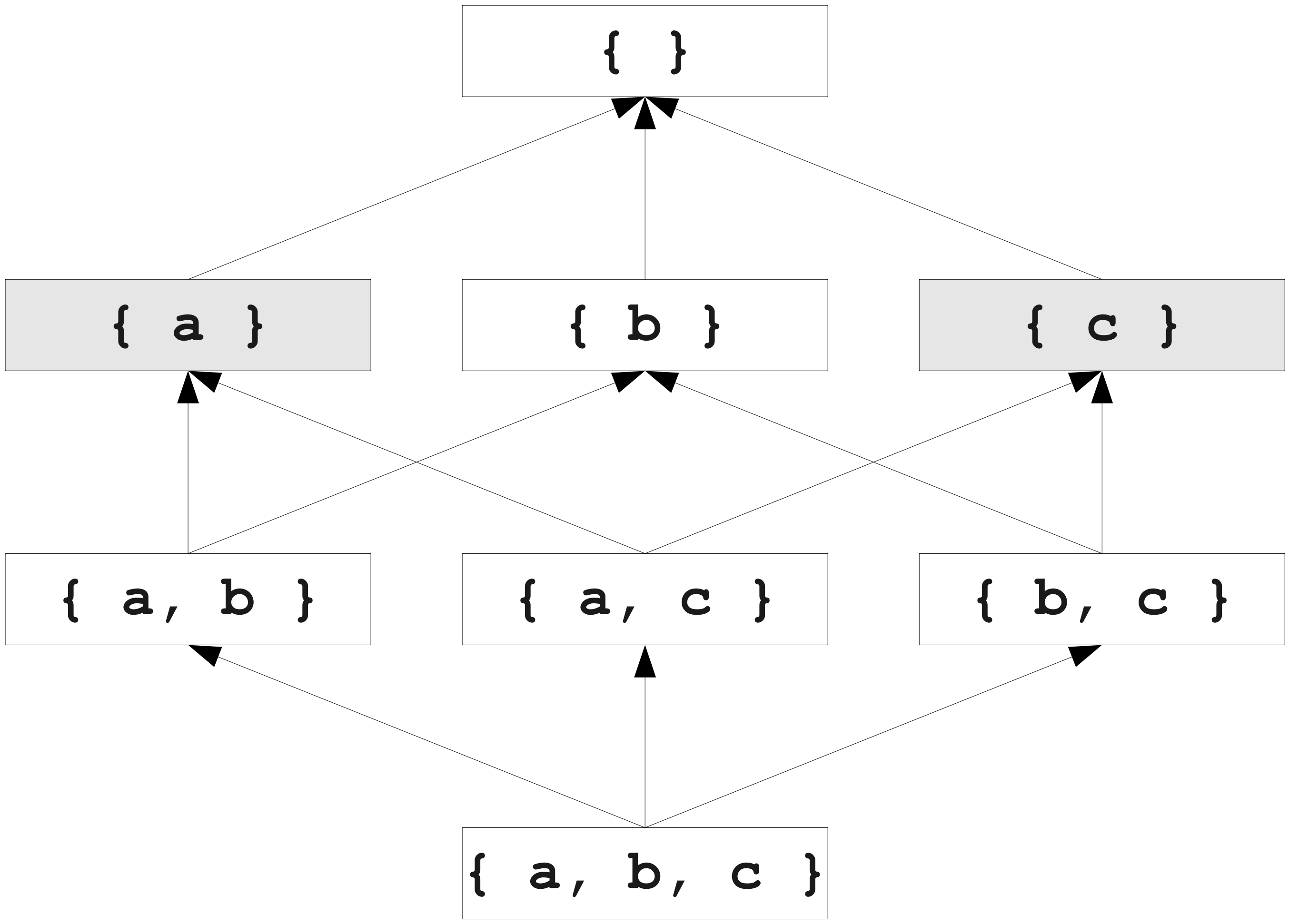

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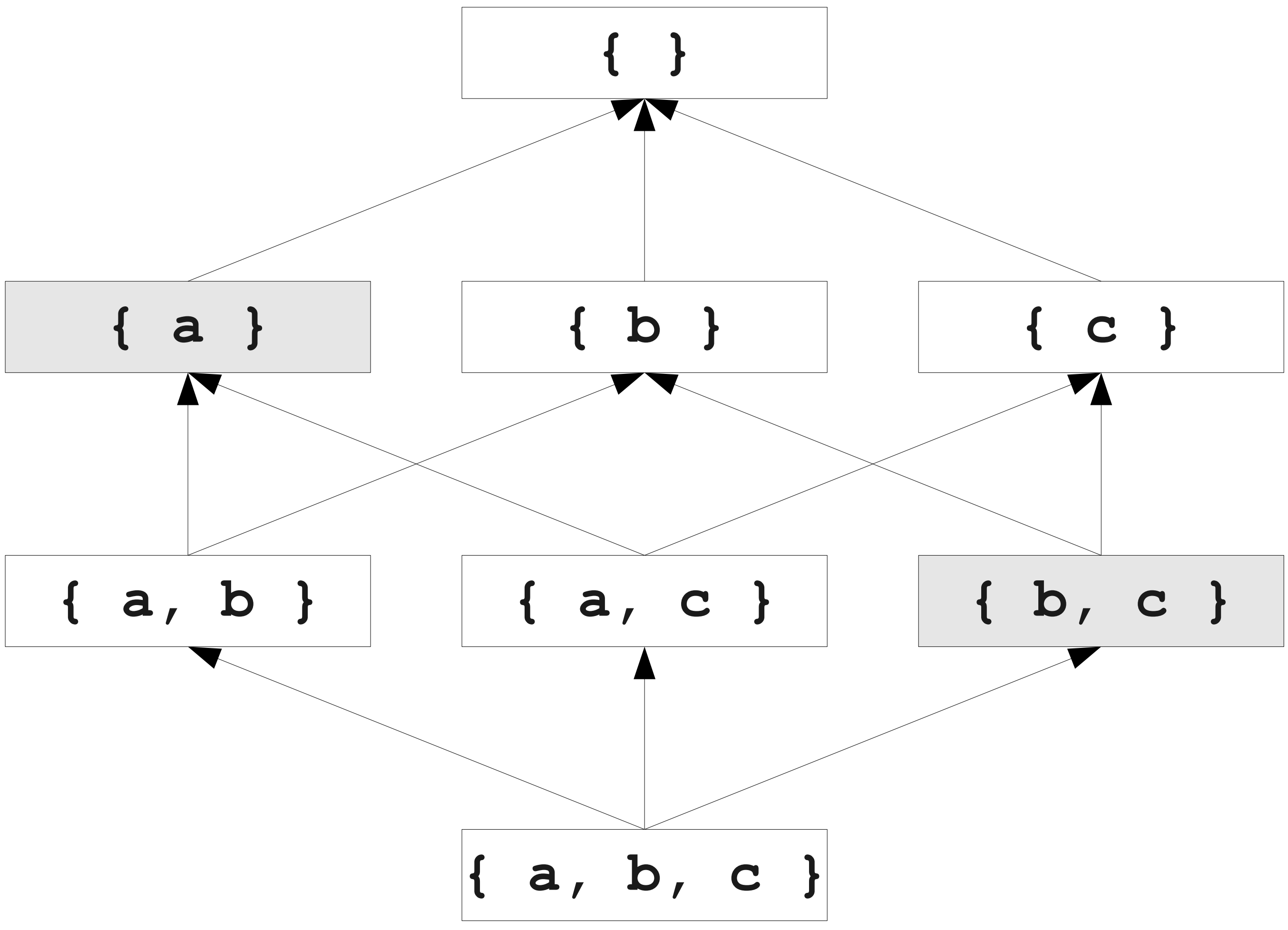


Formal Definitions

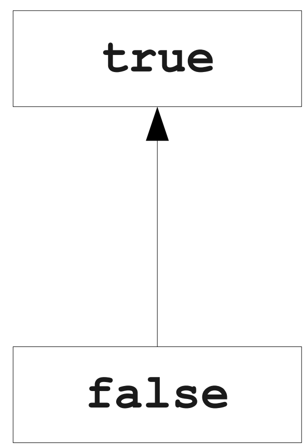
- A **meet semilattice** is a pair $(D, \wedge)$, where
  - D is a domain of elements.
  - $\wedge$ is a **meet operator** that is
    - **idempotent:** $x \wedge x = x$
    - **commutative:** $x \wedge y = y \wedge x$
    - **associative:** $(x \wedge y) \wedge z = x \wedge (y \wedge z)$
- If $x \wedge y = z$, we say that z is the **meet** or (**greatest lower bound**) of x and y.
- Every meet semilattice has a **top element** denoted $\top$ such that $\top \wedge x = x$ for all x.

An Example Semilattice

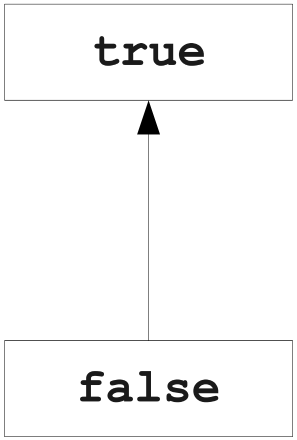
- The set of natural numbers and the **max** function.
- Idempotent
  - **max**{a, a} = a
- Commutative
  - **max**{a, b} = **max**{b, a}
- Associative
  - **max**{a, **max**{b, c}} = **max**{**max**{a, b}, c}
- Top element is 0:
  - **max**{0, a} = a

Is this a Meet Semilattice?

Is this a Meet Semilattice?



Is this a Meet Semilattice?



What is the meet operator here?

Is this a Meet Semilattice?

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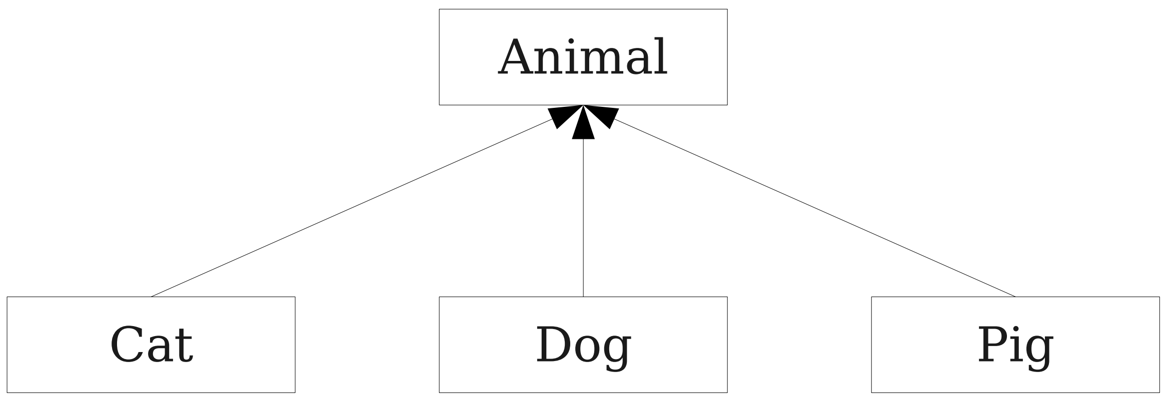

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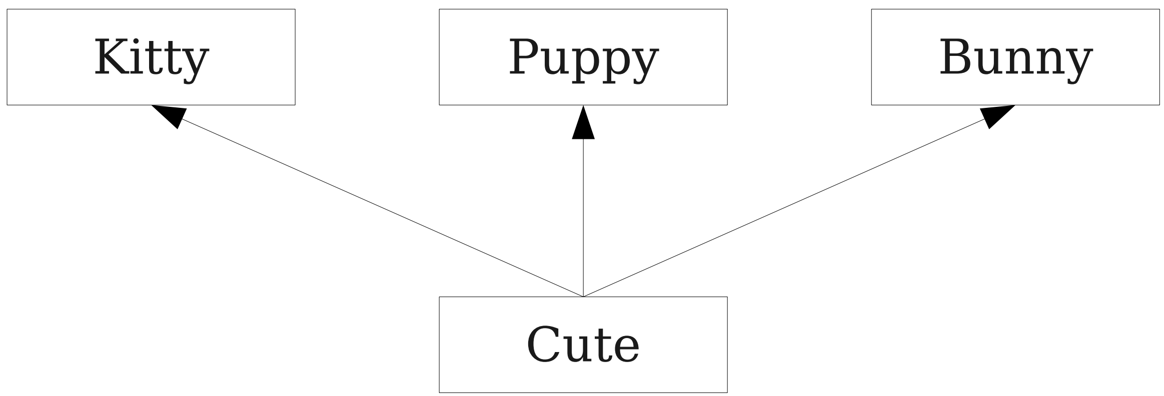

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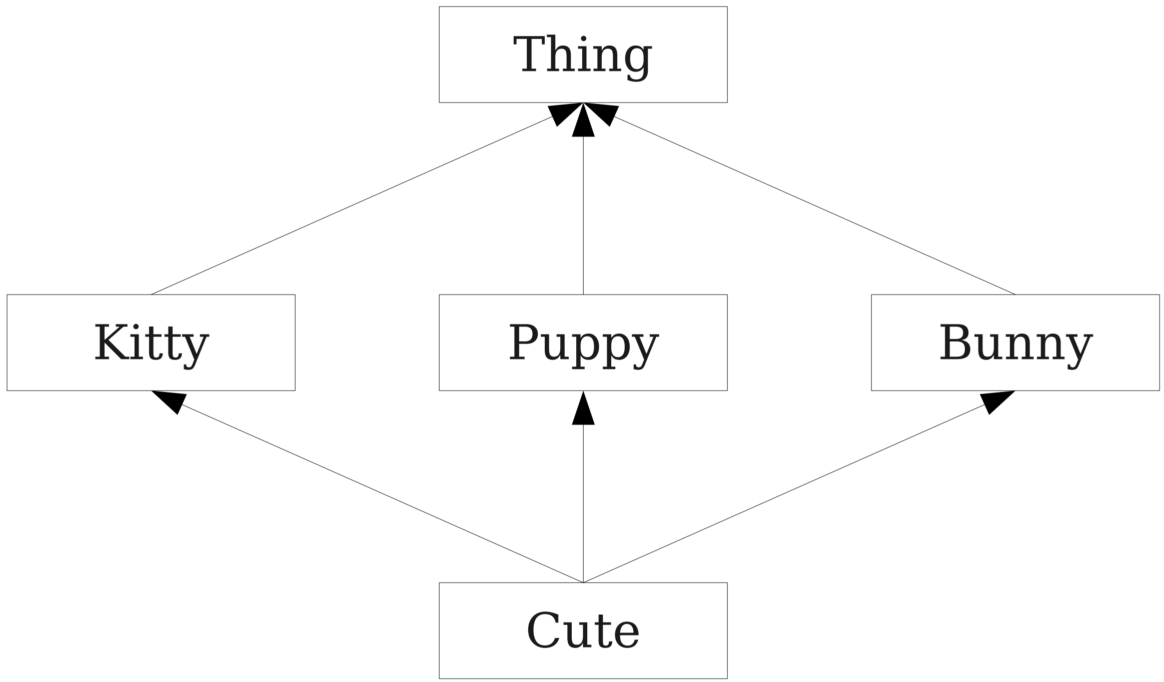


A Semilattice for Liveness

- Sets of live variables and the set union operation.
- Idempotent:
  - $x \cup x = x$
- Commutative:
  - $x \cup y = y \cup x$
- Associative:
  - $(x \cup y) \cup z = x \cup (y \cup z)$
- Top element:
  - The empty set: $\emptyset \cup x = x$

Semilattices and Program Analysis

- Semilattices naturally solve many of the problems we encounter in global analysis.
- How do we combine information from multiple basic blocks?
  - Use the meet of all of those blocks.
- What value do we give to basic blocks we haven't seen yet?
  - Use the top element.
- How do we know that the algorithm always terminates?
  - Actually, we still don't! More on that later.

A General Framework

- A global analysis is a tuple (**D**, **V**, **∧**, **F**, **I**), where
  - **D** is a direction (forward or backward)
    - The order to visit statements **within** a basic block, not the order in which to visit the basic blocks.
  - **V** is a set of values.
  - **∧** is a meet operator over those values.
  - **F** is a set of transfer functions $f : \mathbf{V} \rightarrow \mathbf{V}$
  - **I** is an initial value.
- The only difference from local analysis is the introduction of the meet operator.

Running Global Analyses

- Assume that (**D**, **V**, **∧**, **F**, **I**) is a forward analysis.
- Set OUT[**s**] = ⊤ for all statements **s**.
- Set OUT[**begin**] = **I**.
- Repeat until no values change:
  - For each statement **s** with predecessors $\mathbf{p_1}$, $\mathbf{p}_2$, … , $\mathbf{p_n}$:
    - Set IN[**s**] = OUT[$\mathbf{p_1}$] ∧ OUT[$\mathbf{p_2}$] ∧ **…** ∧ OUT[$\mathbf{p_n}$]
    - Set OUT[**s**] = f_s (IN[**s**])
- The order of this iteration does not matter.

For Comparison

- Set IN[**s**] = $\top$ for all statement **s**.
- Set IN[**exit**] = I.
- Repeat until no changes occur:
  - For each statement **s**:
    - Set OUT[**s**] = IN[$\boldsymbol{x}_1$]$\wedge\ldots\wedge$IN[$\boldsymbol{x}_n$] where $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n$ are successors of **s**.
    - Set IN[**s**] = f_s (OUT[**s**])

- Set IN[**s**] = { } for each statement **s**.
- Set IN[**exit**] to the set of variables known to be live on exit.
- Repeat until no changes occur:
  - For each statement **s** of the form **a** = **b** + **c**:
    - Set OUT[**s**] to set union of IN[$\boldsymbol{x}$] for each successor $\boldsymbol{x}$ of **s**.
    - Set IN[**s**] to (OUT[**s**] – **a**) ∪ {**b**, **c**}.

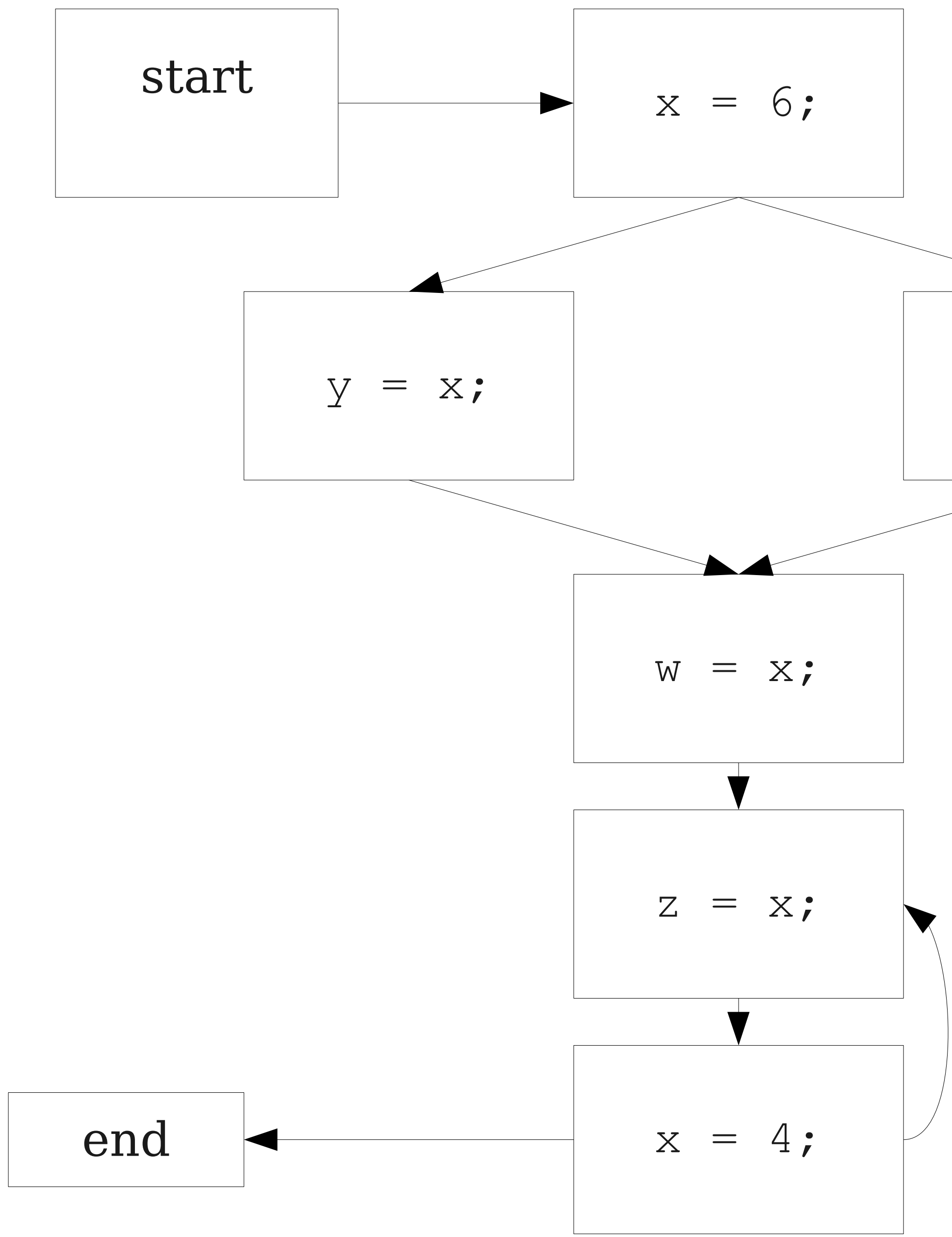
The Dataflow Framework

- This form of analysis is called the **dataflow framework**.
- Can be used to easily prove an analysis is sound.
- With certain restrictions, can be used to prove that an analysis eventually terminates.
  - Again, more on that later.

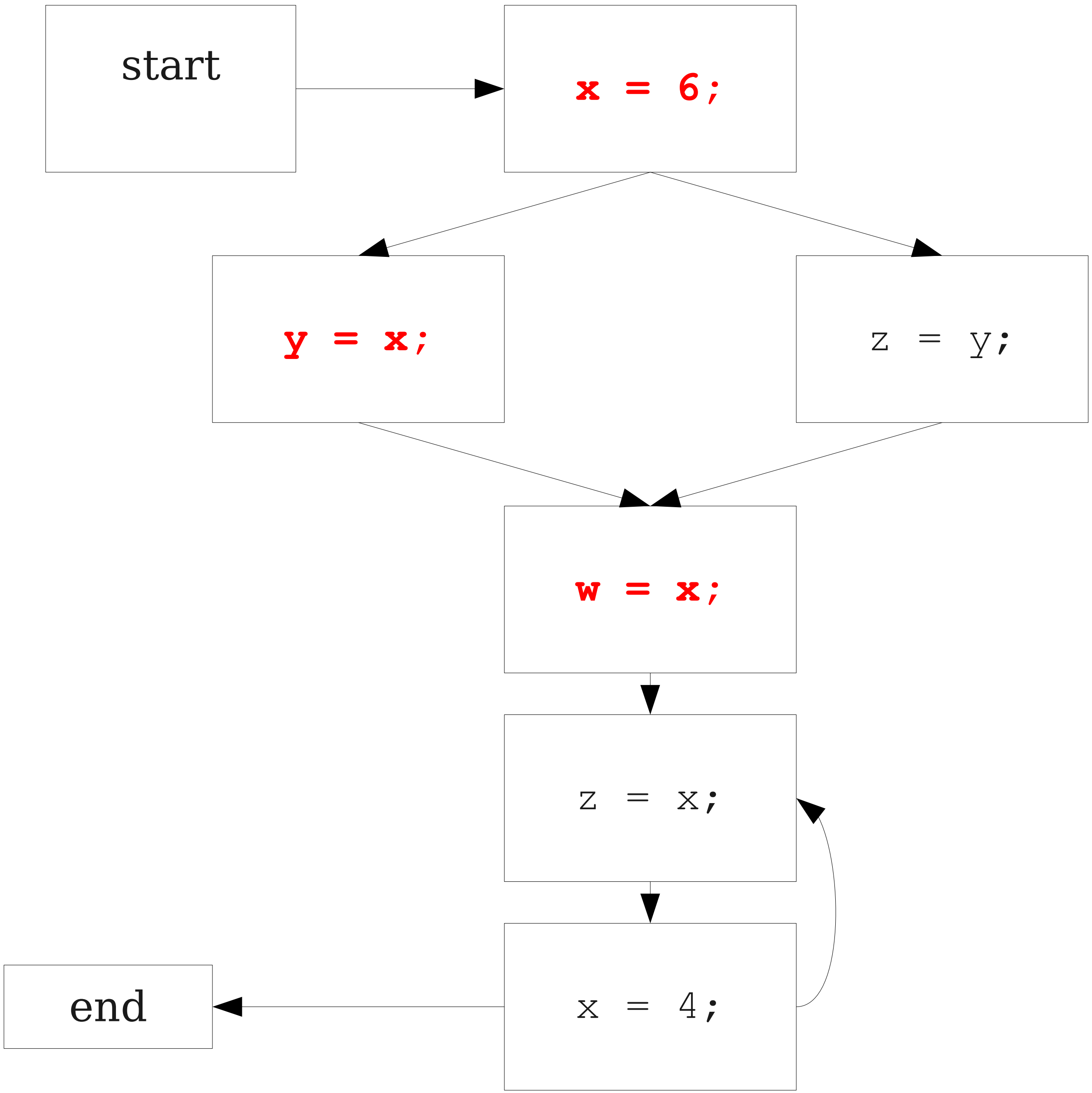
Global Constant Propagation

- **Constant propagation** is an optimization that replaces each variable that is known to be a constant value with that constant.
- An elegant example of the dataflow framework.

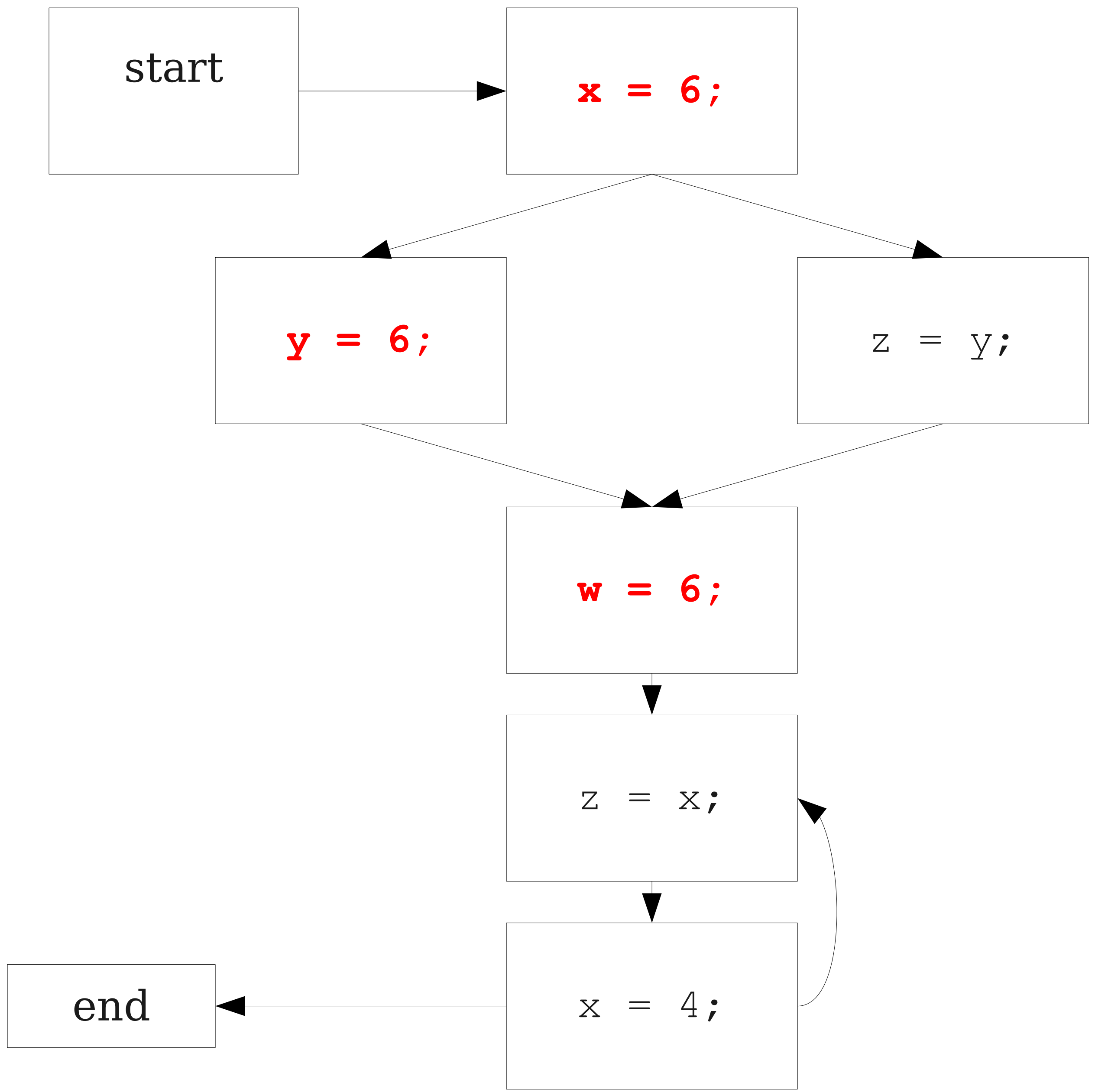
Global Constant Propagation



Global Constant Propagation



Global Constant Propagation



Constant Propagation Analysis

- In order to do a constant propagation, we need to track what values might be assigned to a variable at each program point.
- Every variable will either
  - Never have a value assigned to it,
  - Have a single constant value assigned to it,
  - Have two or more constant values assigned to it, or
  - Have a known non-constant value.
- Our analysis will propagate this information throughout a CFG to identify locations where a value is constant.

Properties of Constant Propagation

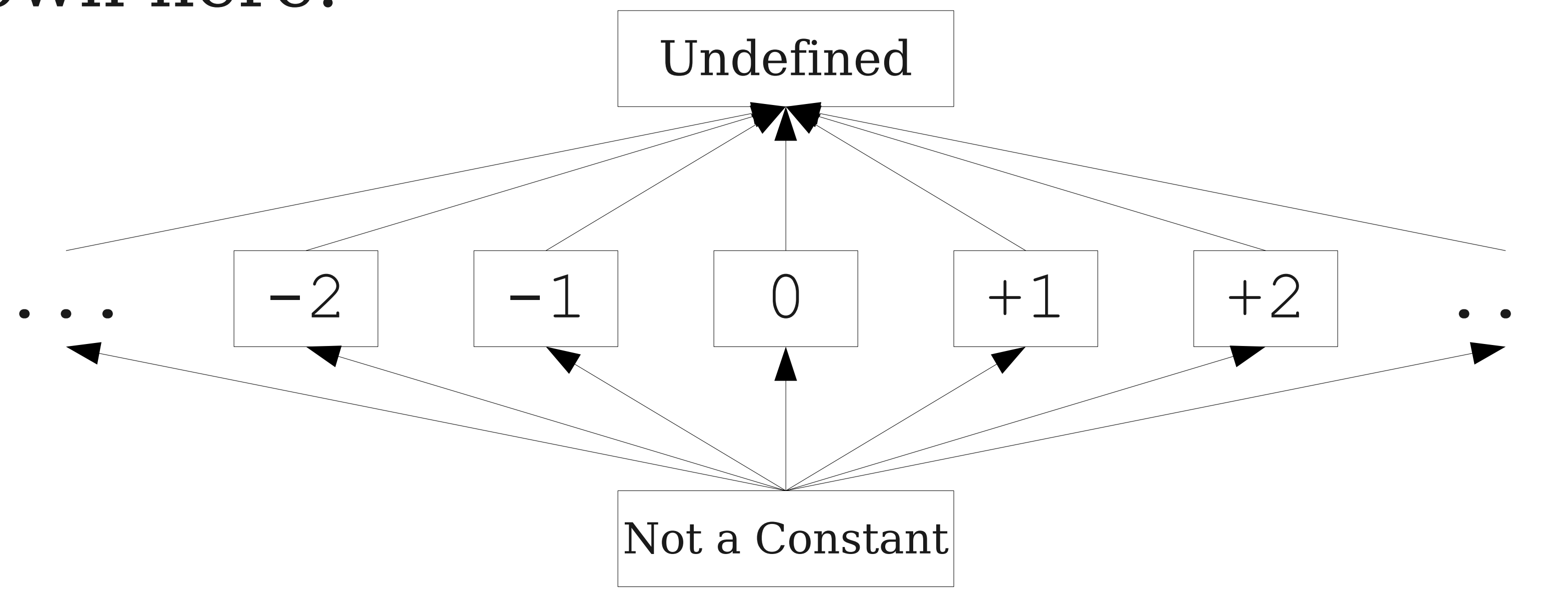
- For now, consider just some single variable **x**.
- At each point in the program, we know one of three things about the value of **x**:
  - **x** is definitely not a constant, since it's been assigned two values or assigned a value that we know isn't a constant.
  - **x** is definitely a constant and has value **k**.
  - We have never seen a value for **x**.
- Note that the first and last of these are **not** the same!
  - The first one means that there may be a way for **x** to have multiple values.
  - The last one means that **x** never had a value at all.

Defining a Meet Operator

- The meet of any two different constants is **Not a Constant**.
  - (If the variable might have two different values on entry to a statement, it cannot be a constant.)
- The meet of **Not a Constant** and any other value is **Not a Constant**.
  - (If on some path the value is known not to be a constant, then on entry to a statement its value can't possibly be a constant.)
- The meet of **Undefined** and any other value is that other value.
  - (If **x** has no value on some path and does have a value on some other path, we can just pretend it always had the assigned value.)

A Semilattice for Constant Propagation

- One possible semilattice for this analysis is shown here:



A Semilattice for Constant Propagation

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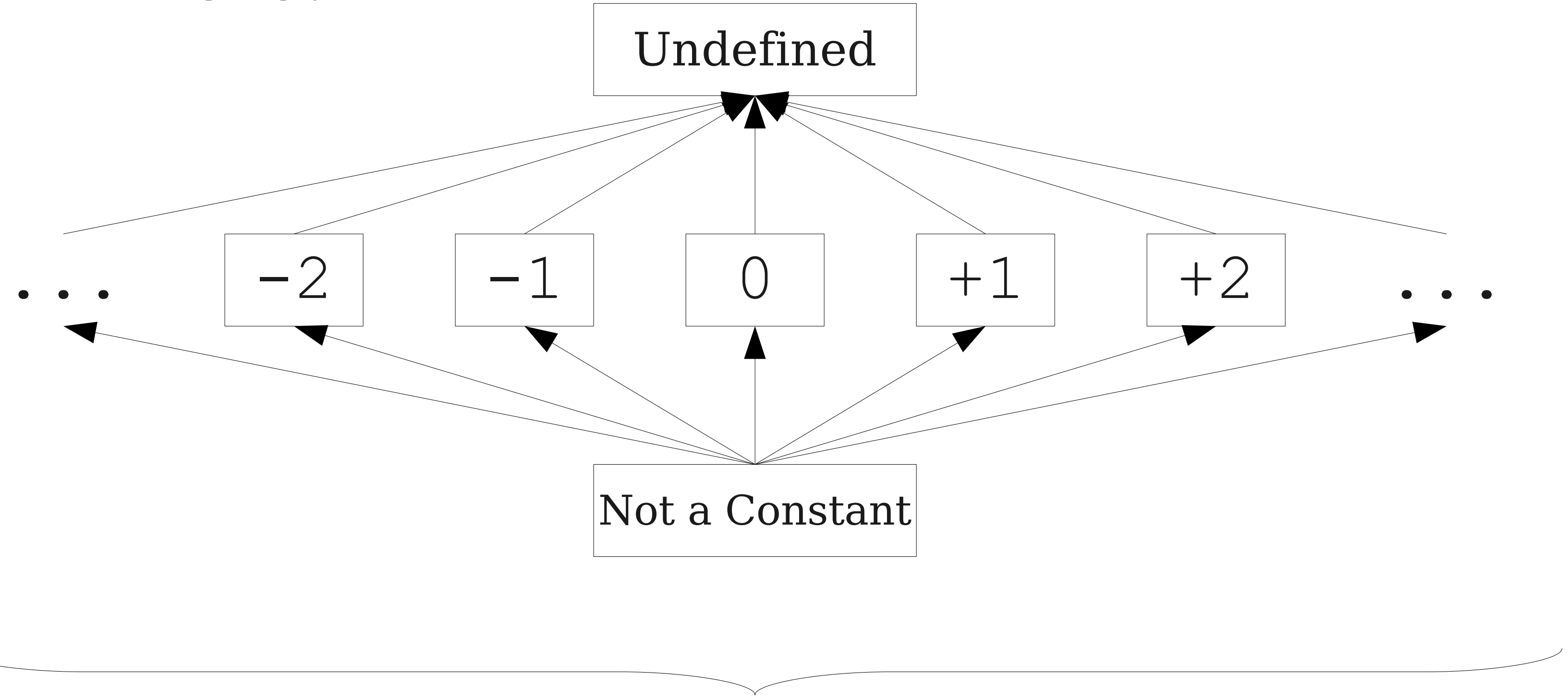

This lattice is infinitely wide!

A Semilattice for Constant Propagation

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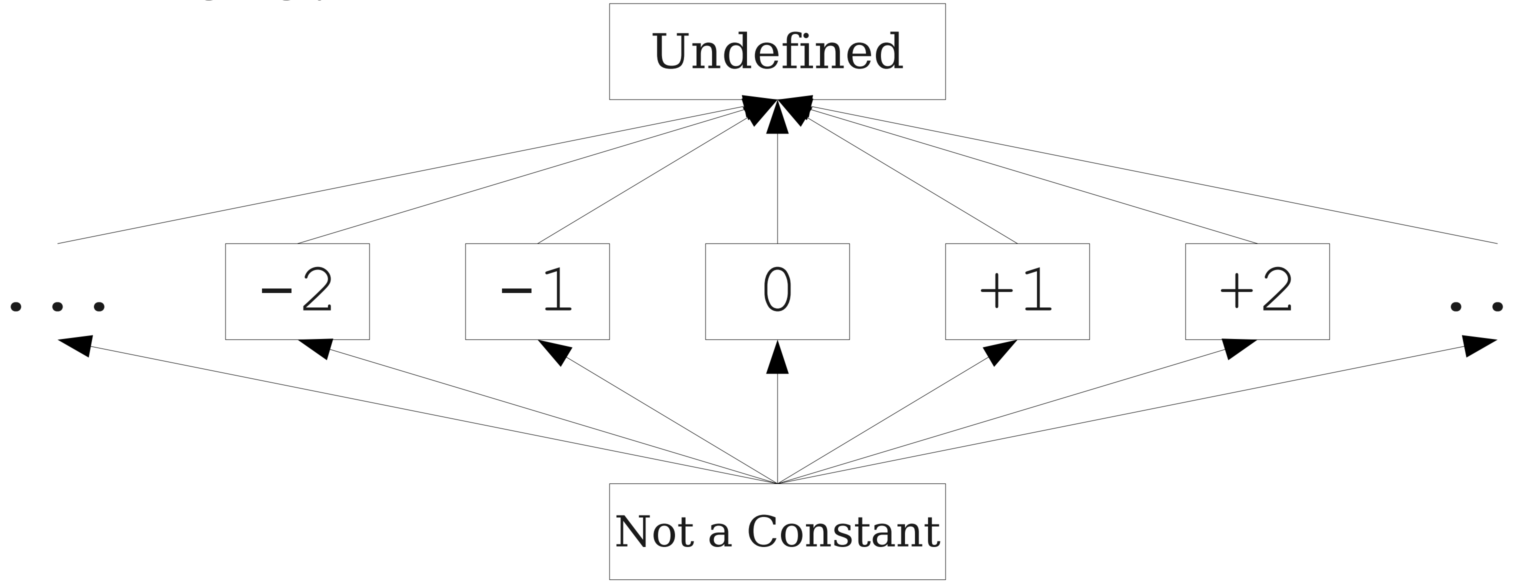

A Semilattice for Constant Propagation

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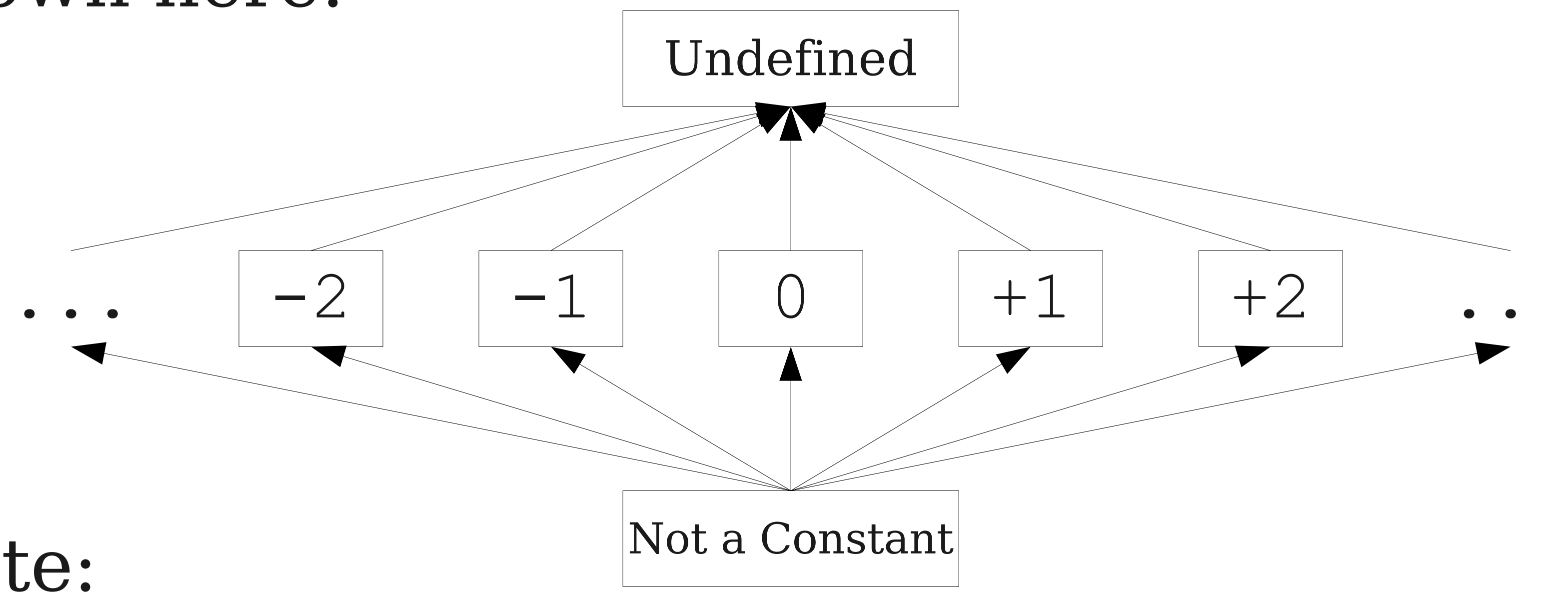
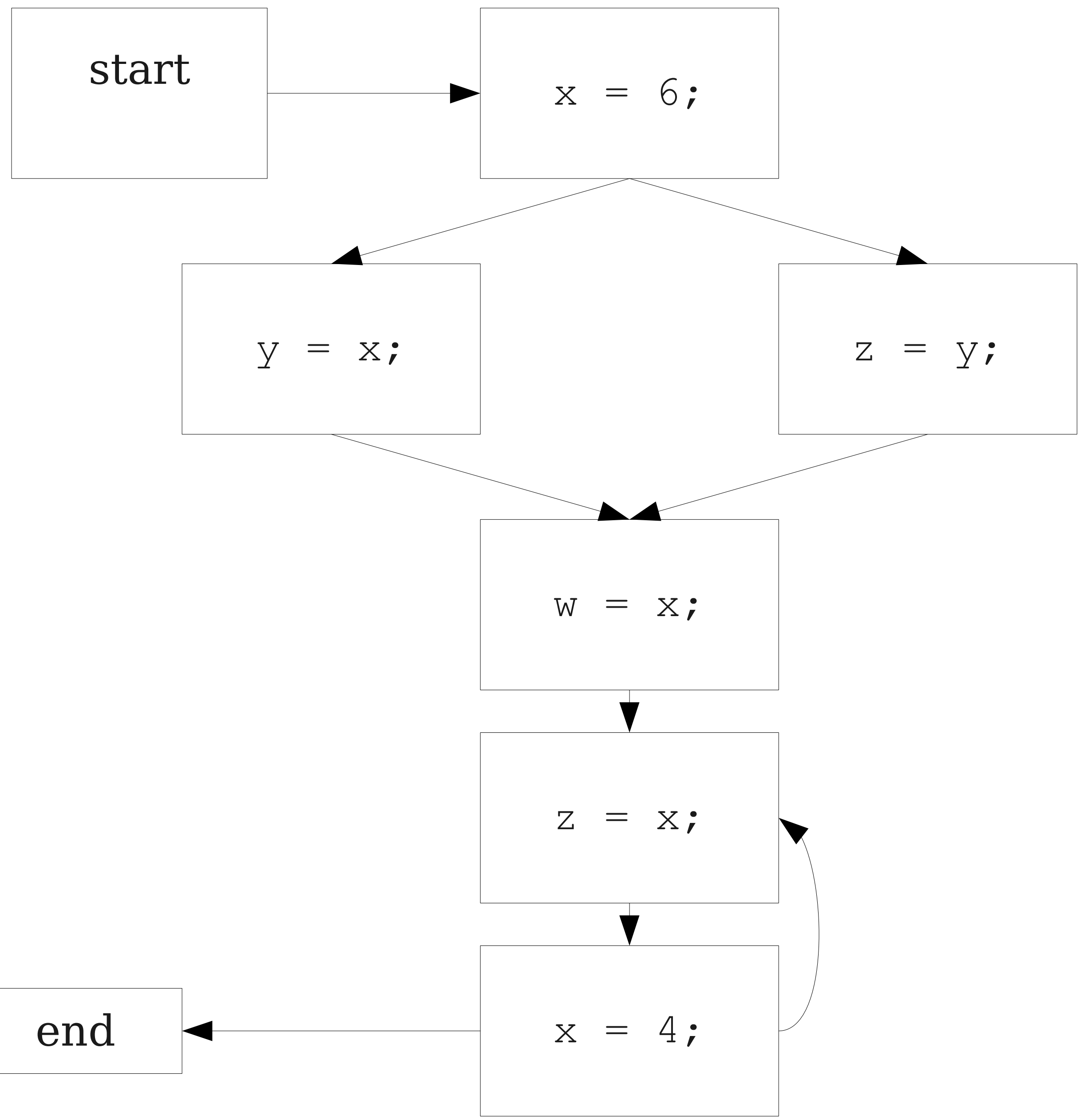
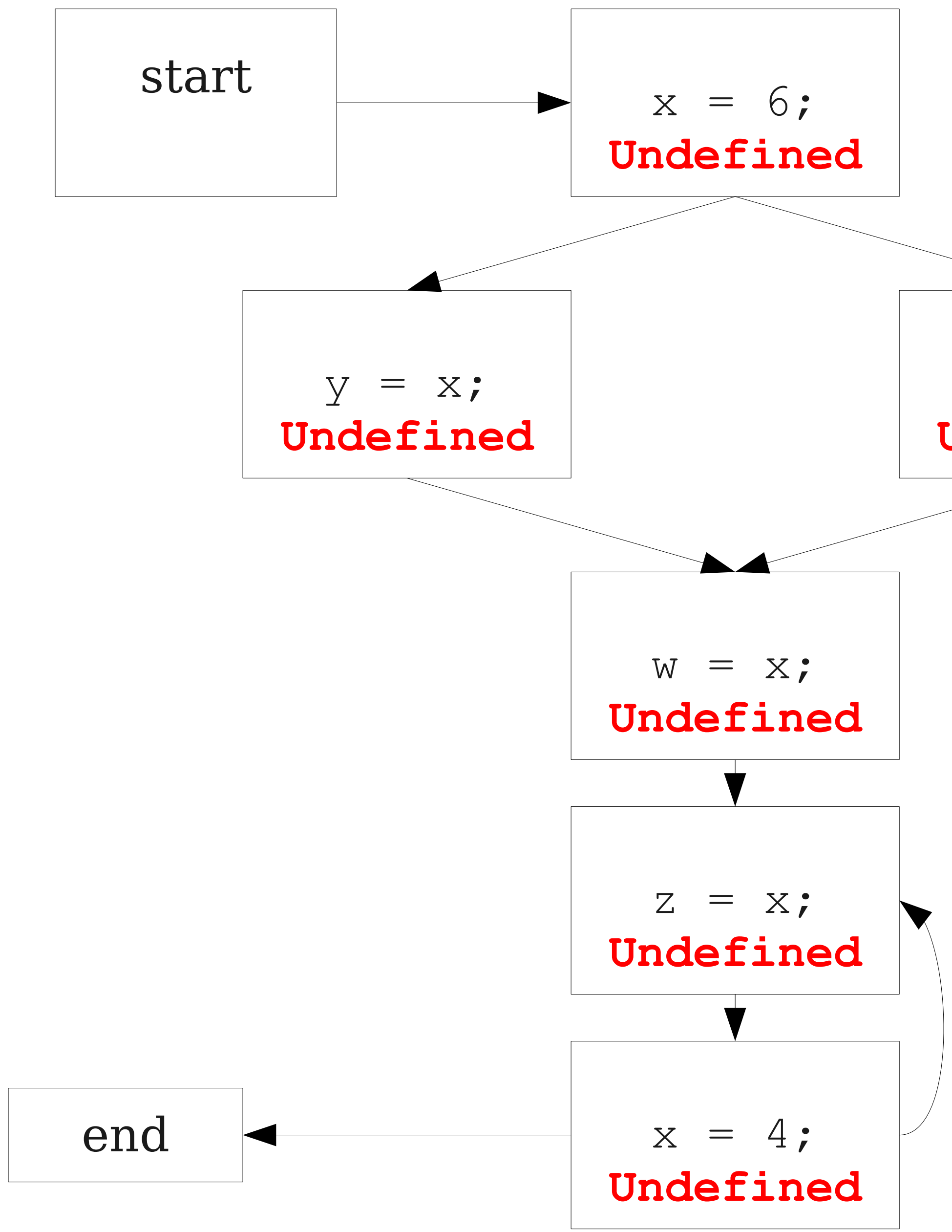


- Note:
  - The meet of any two different constants is **Not a Constant**.
  - The meet of **Undefined** and any value is that value.
  - The meet of **Not a Constant** and any value is **Not a Constant**.

Global Constant Propagation



Global Constant Propagation



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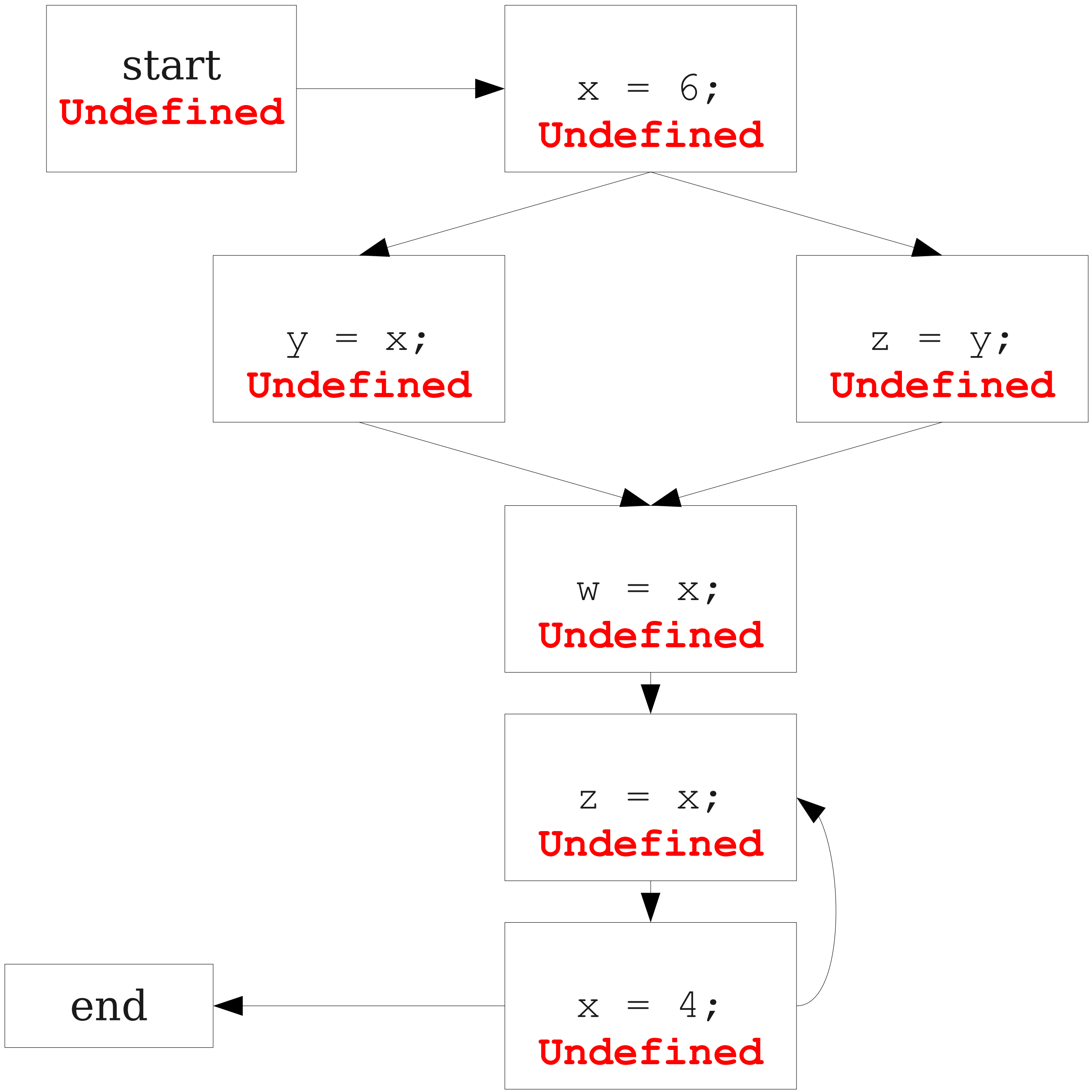

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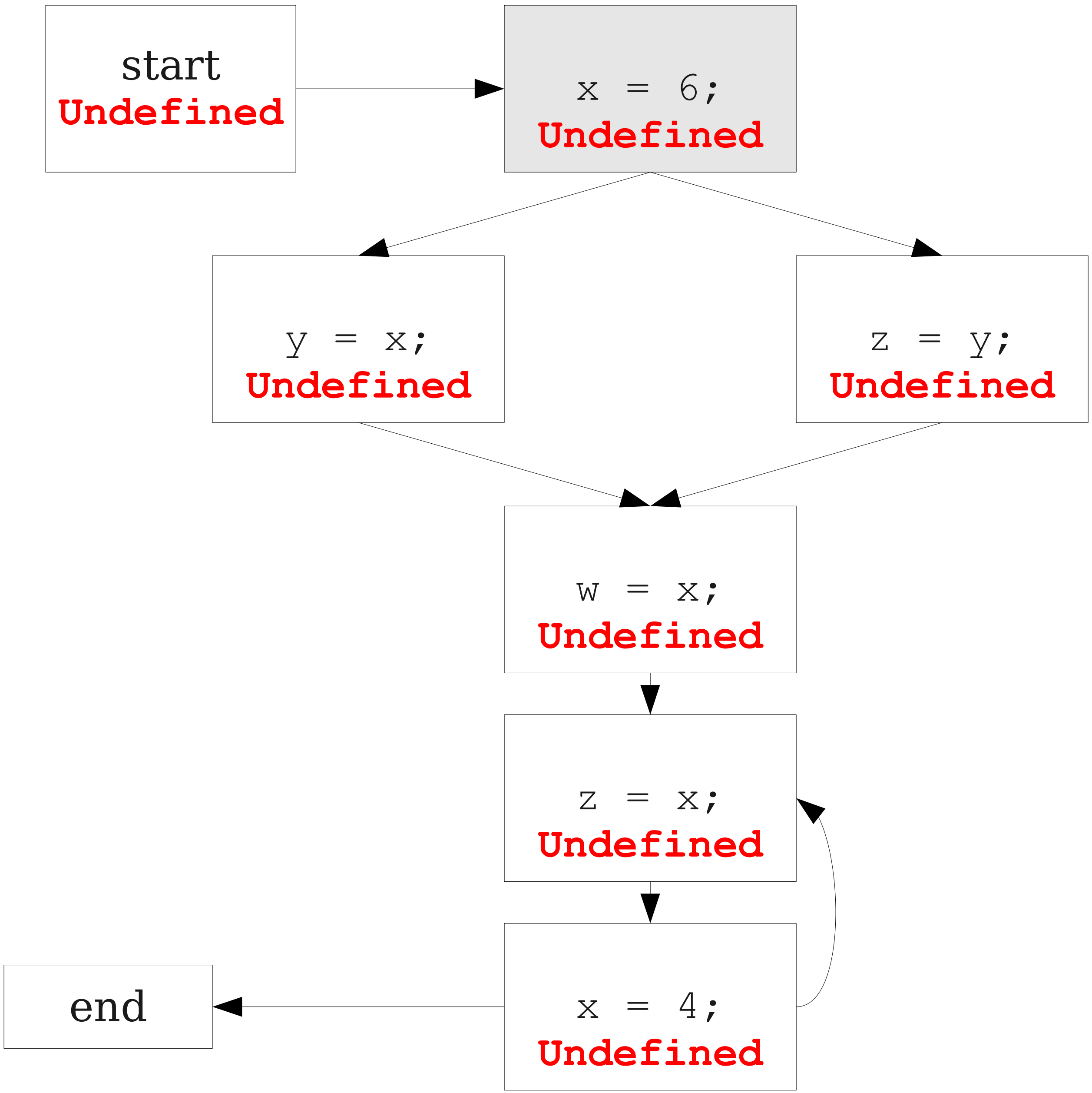

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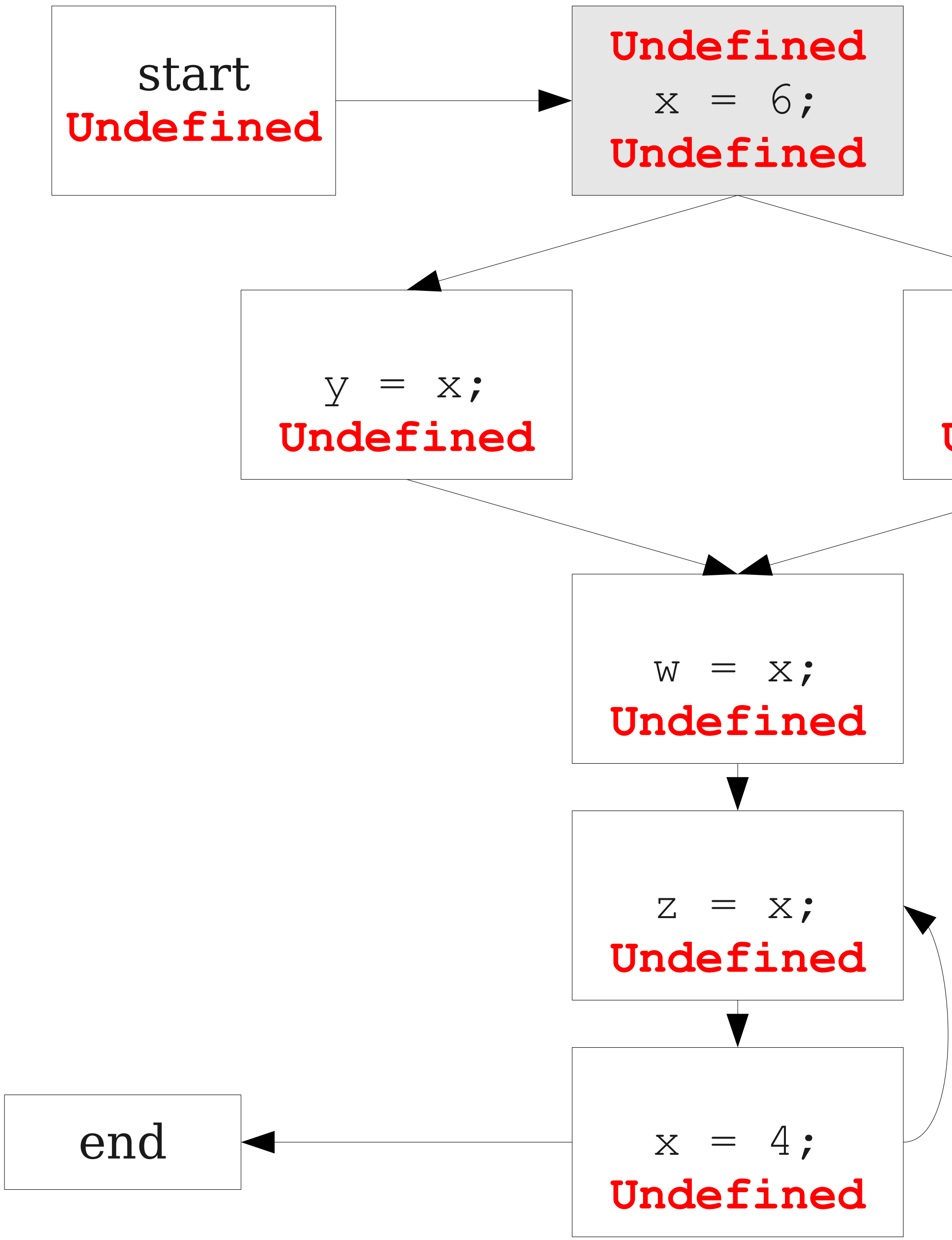

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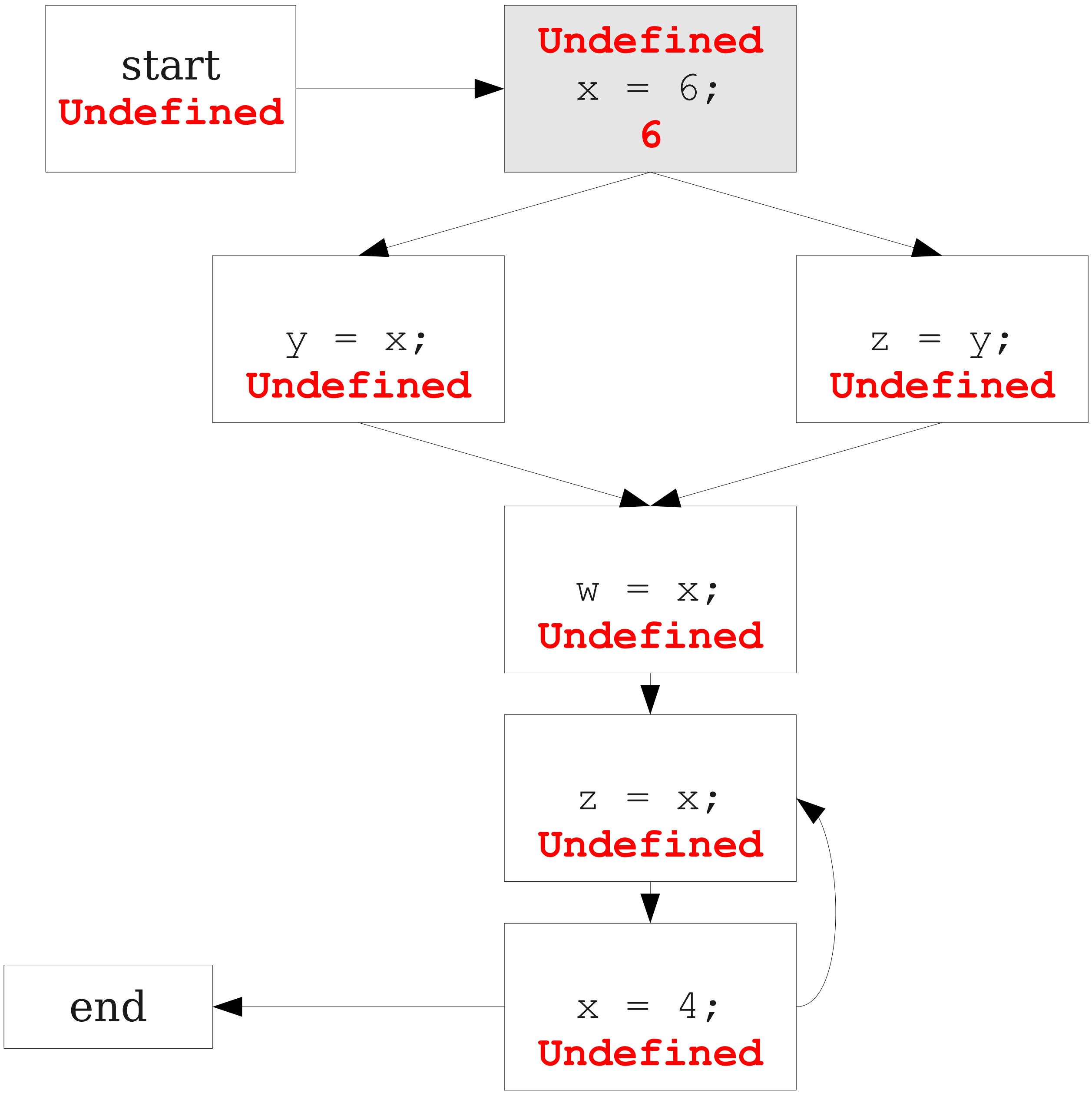

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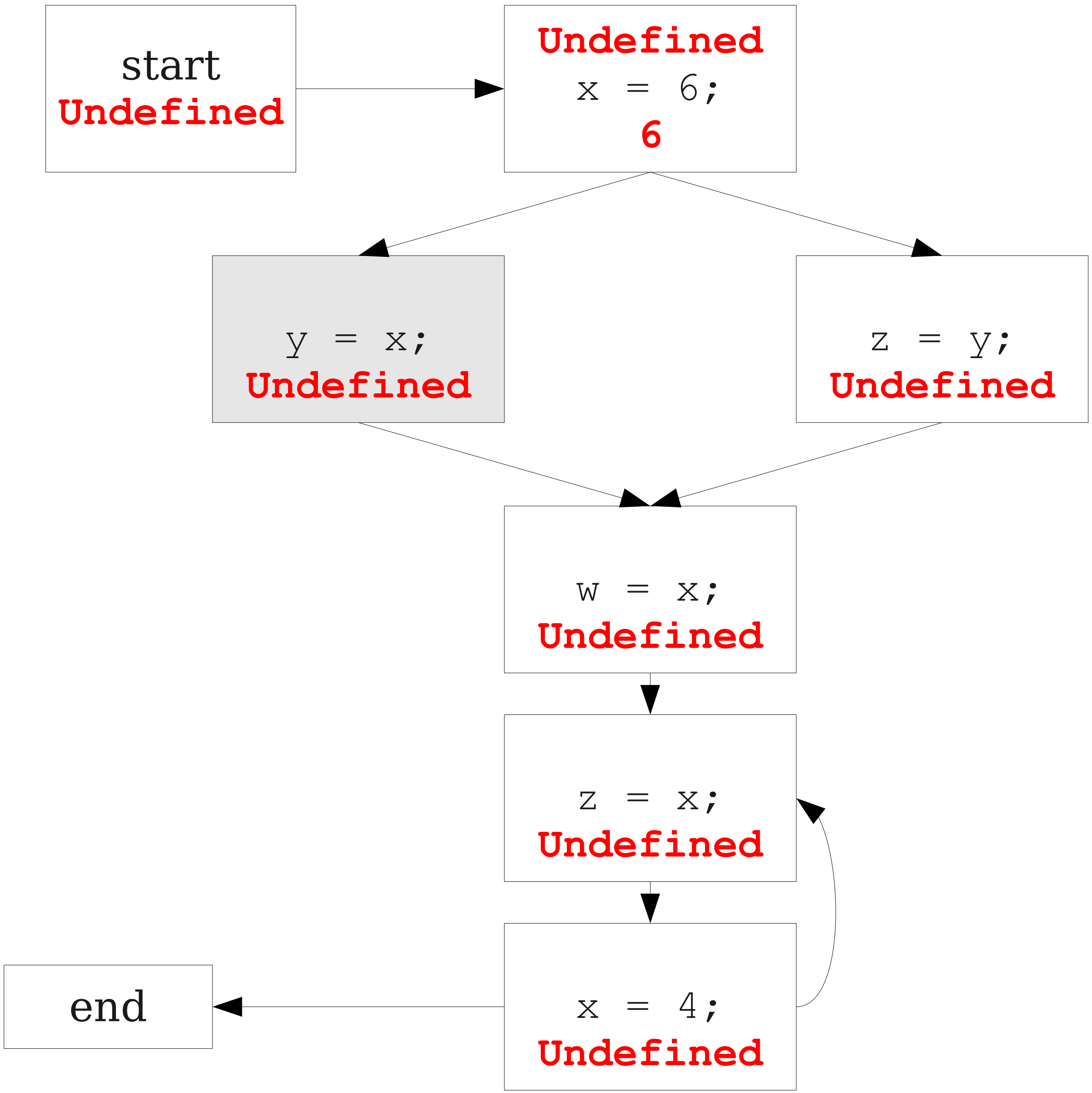
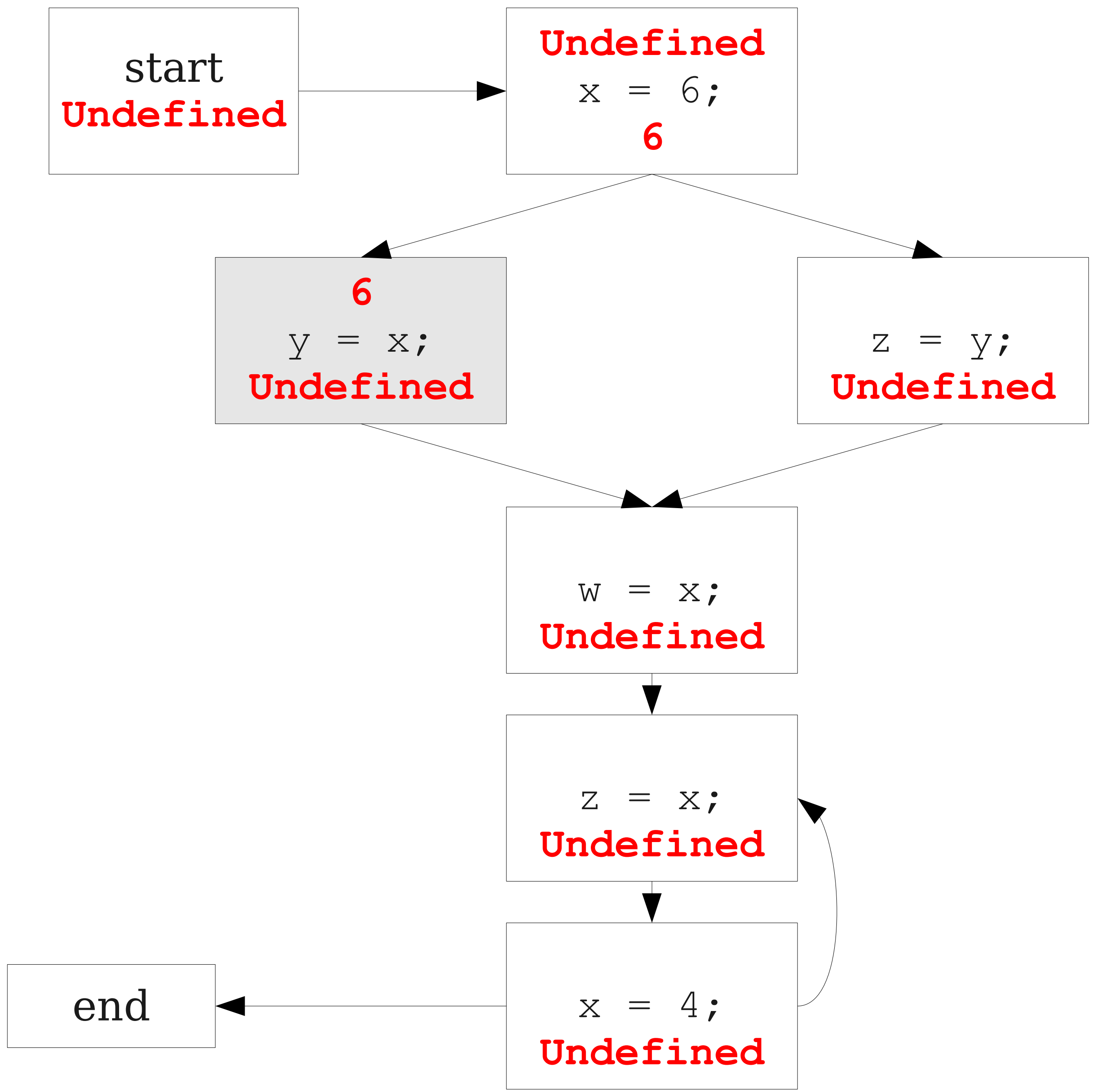


Global Constant Propagation



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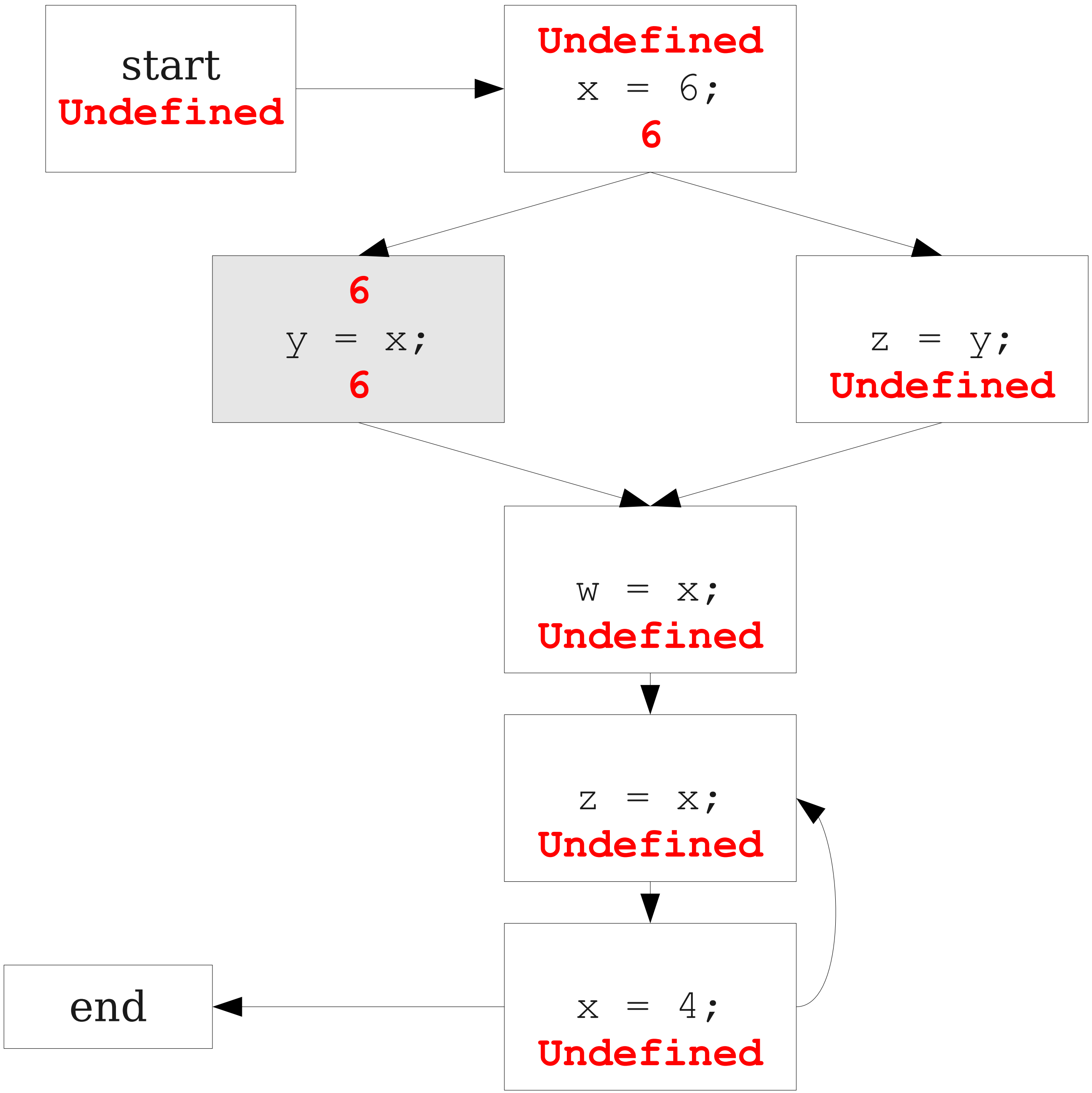

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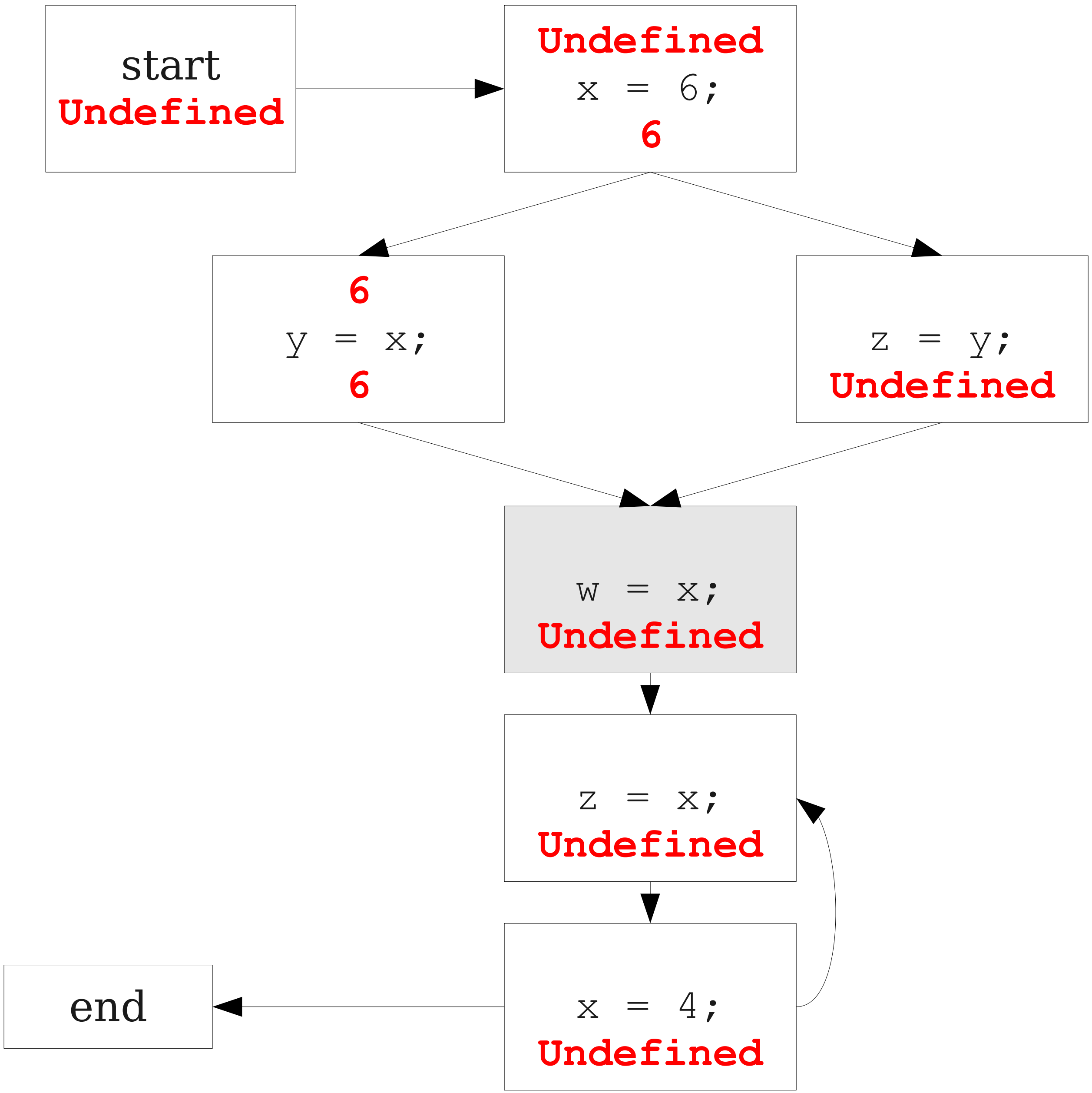

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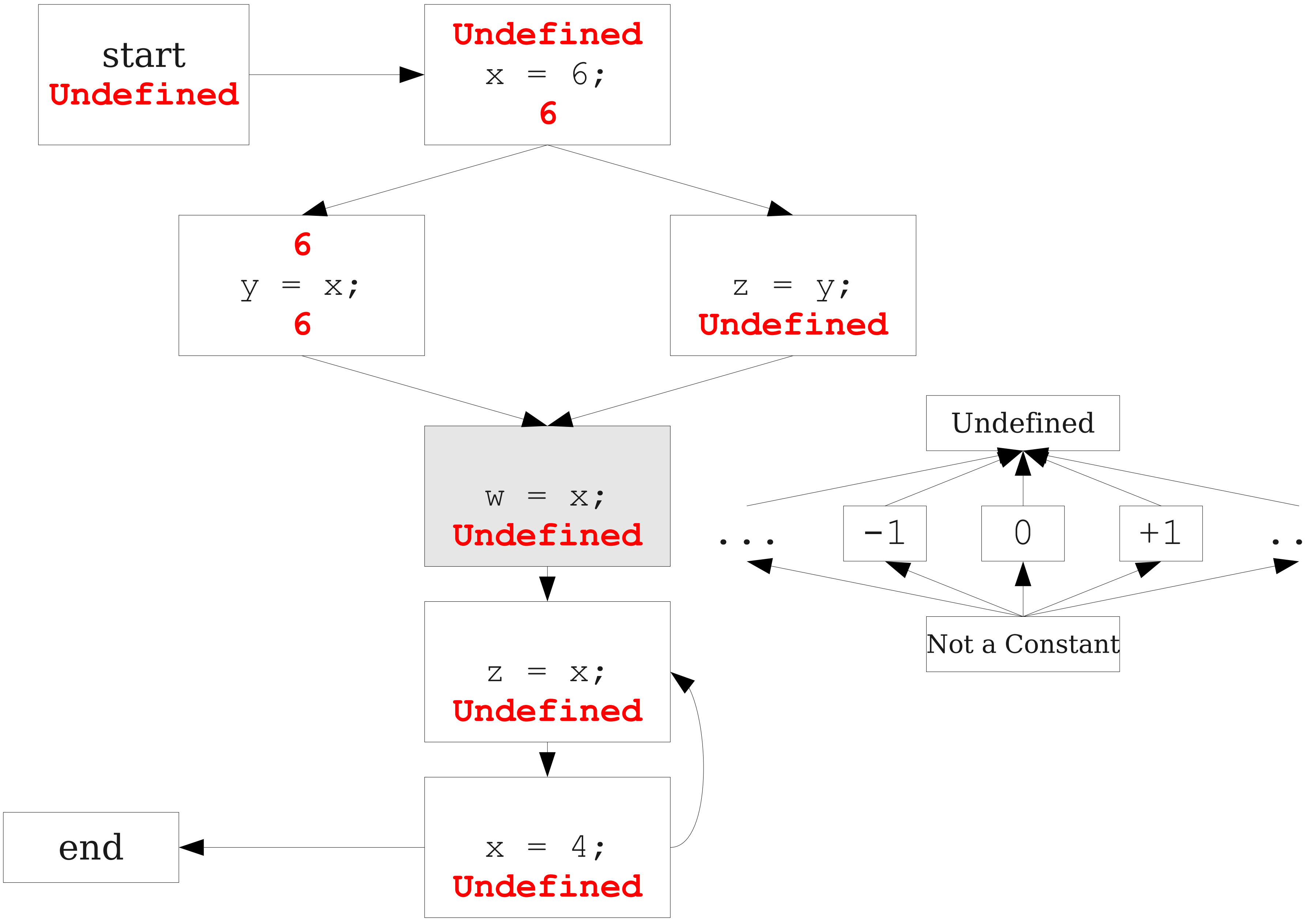

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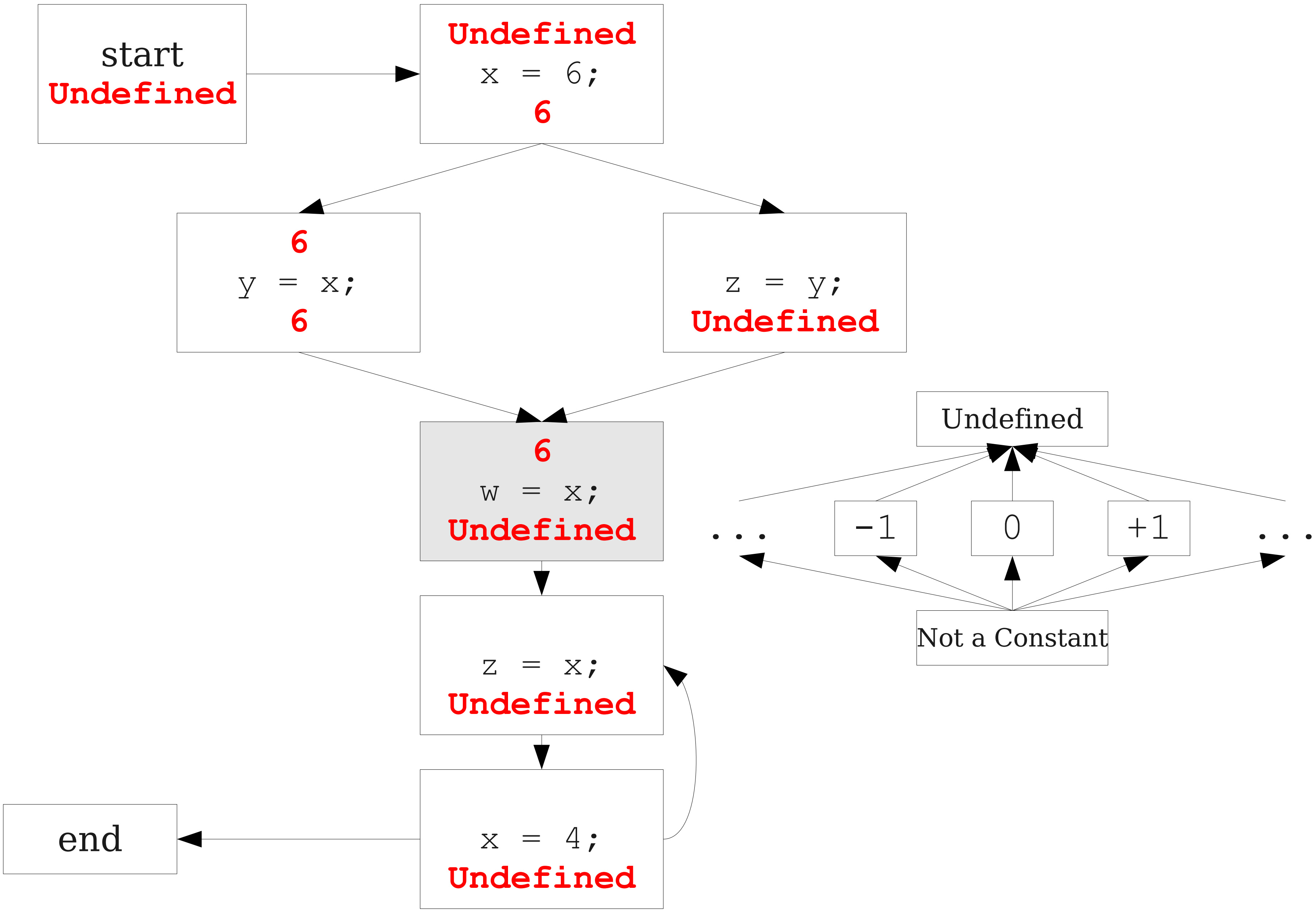

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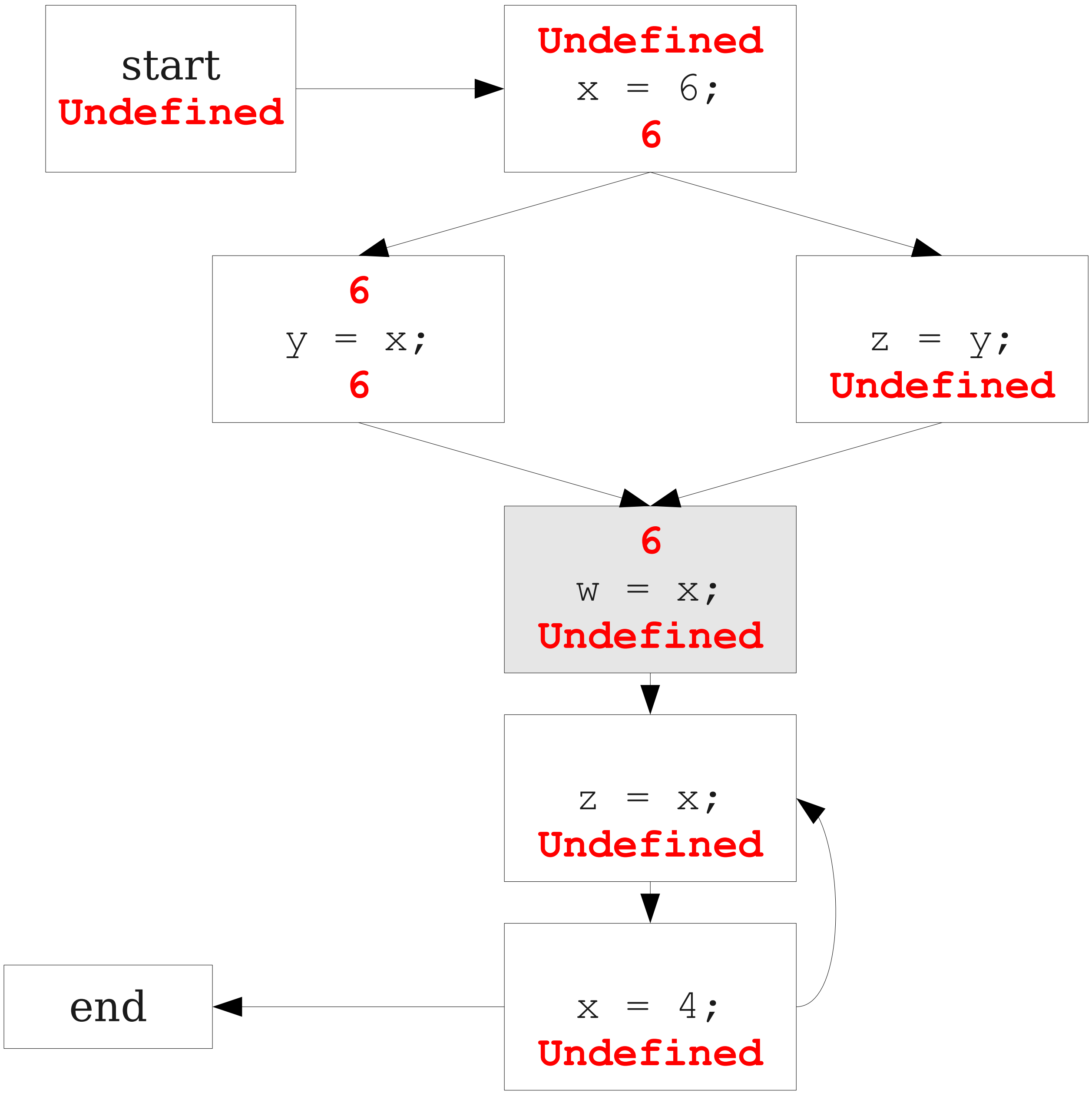

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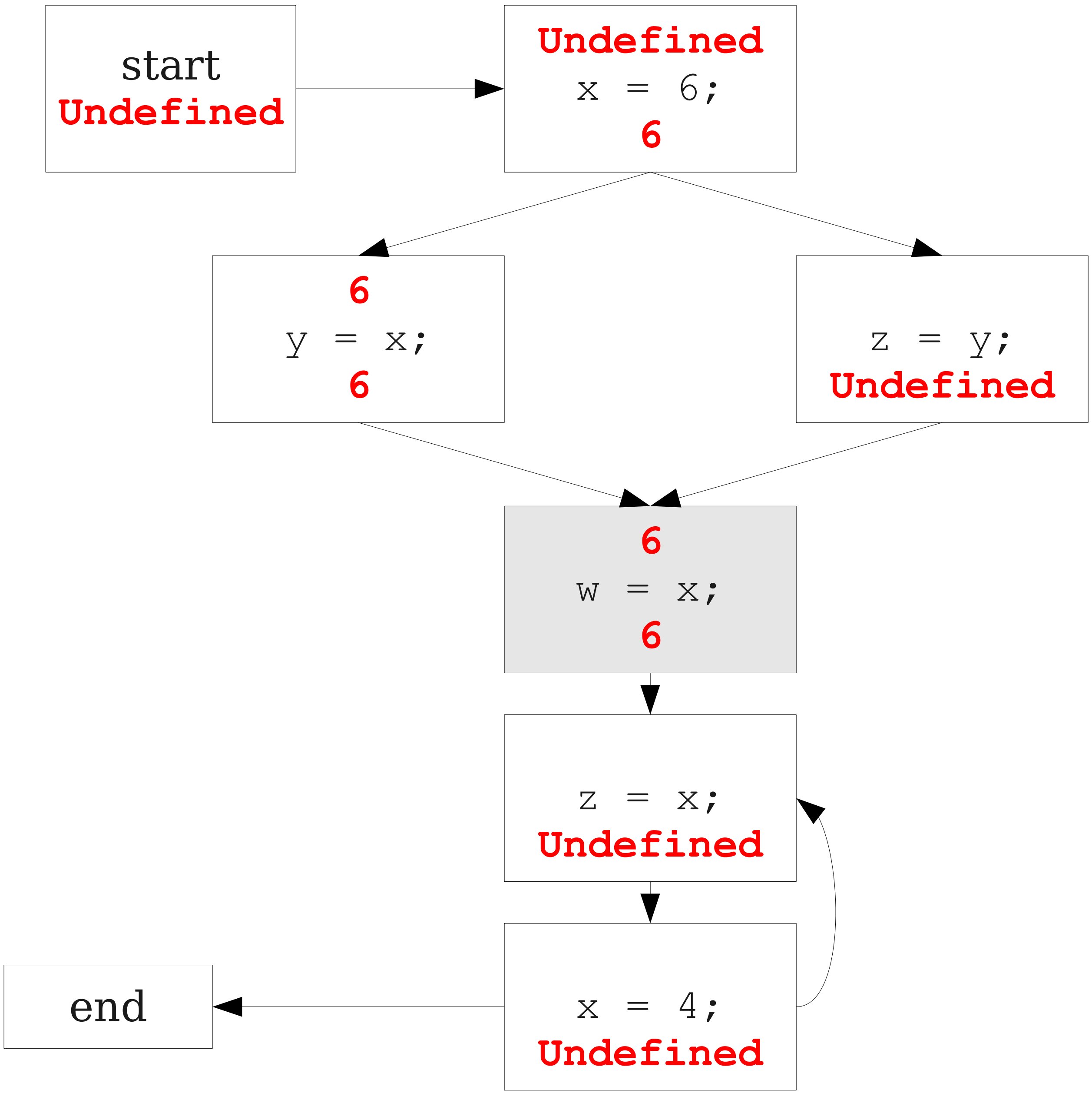

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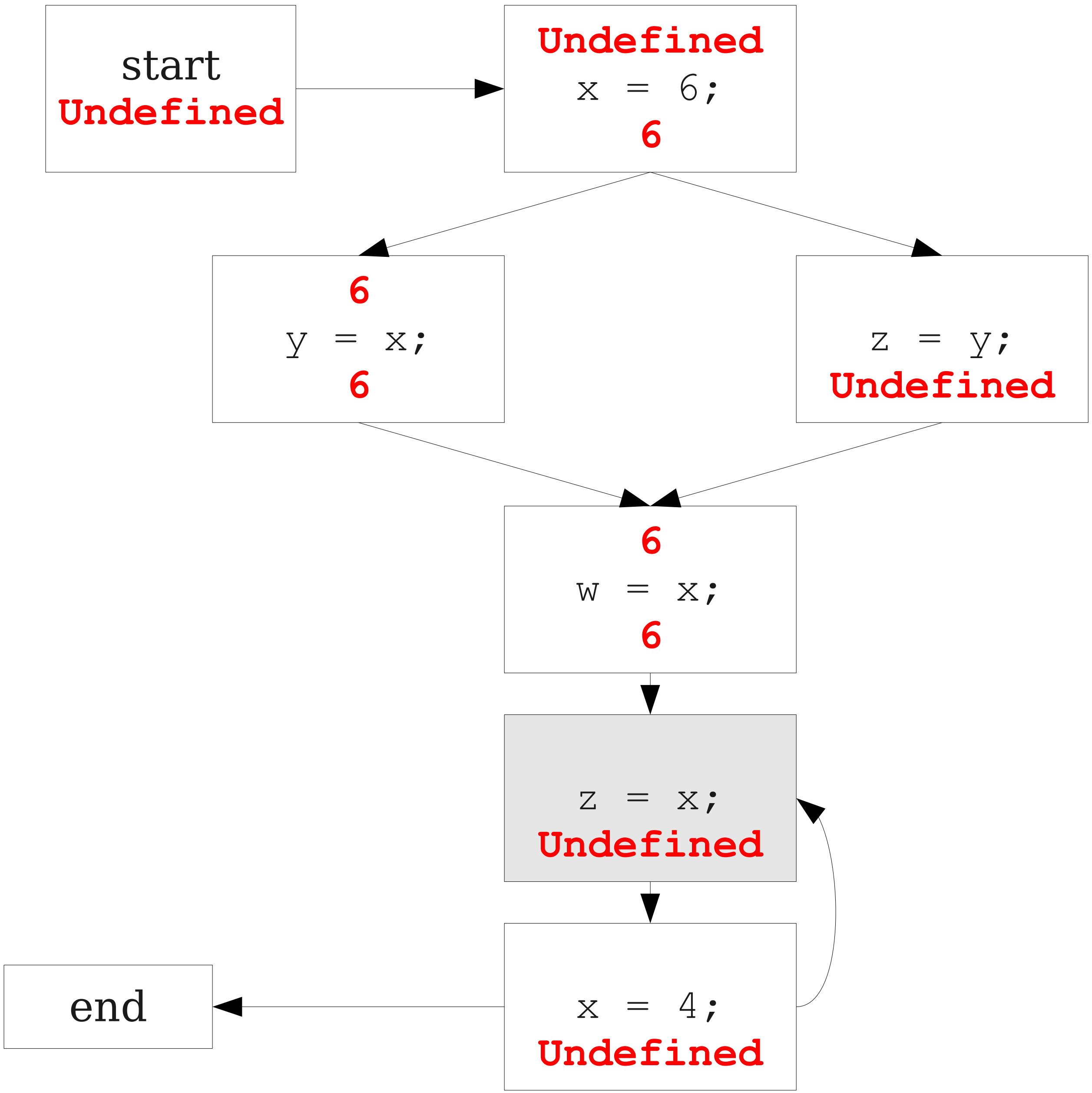

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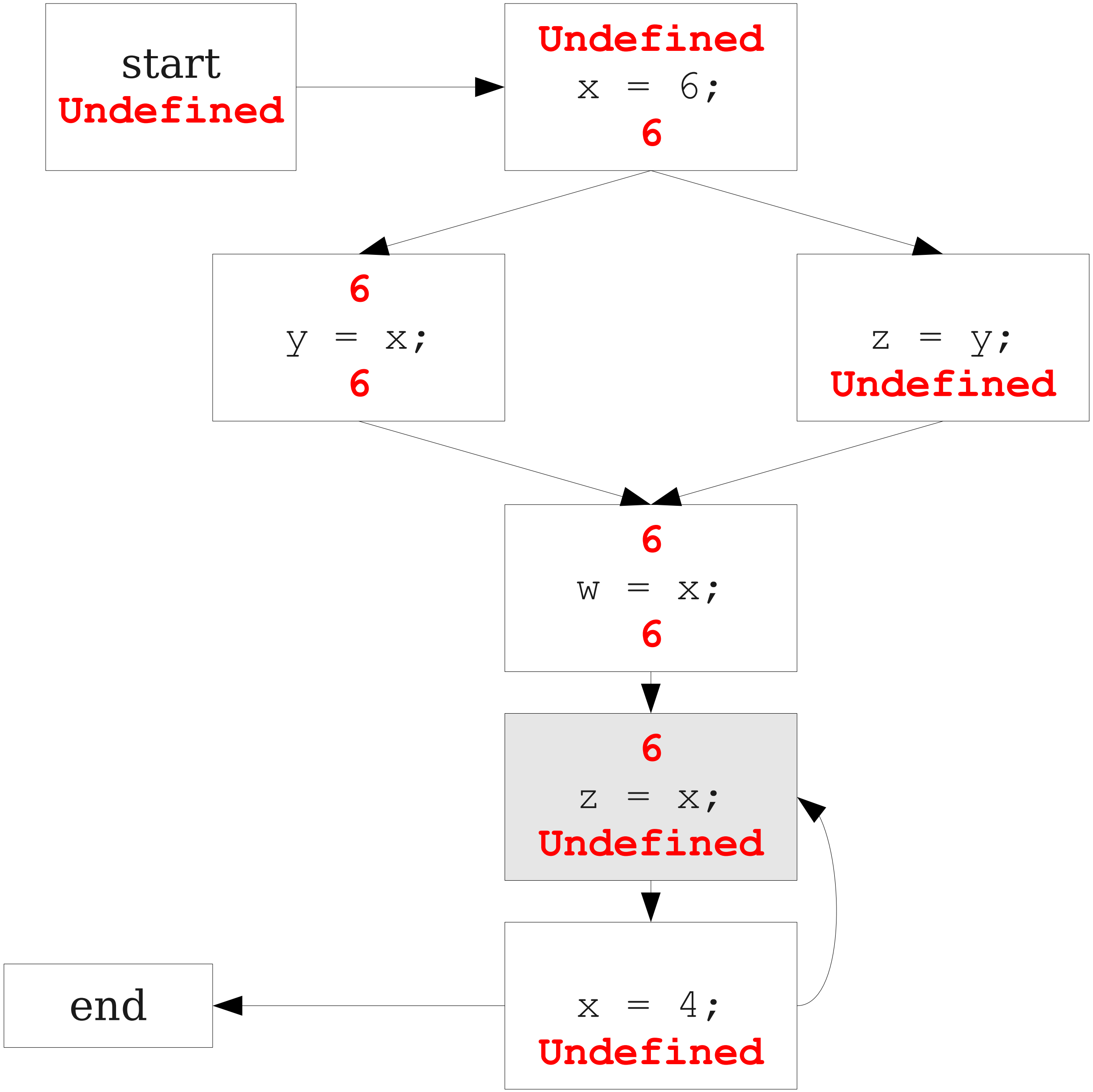

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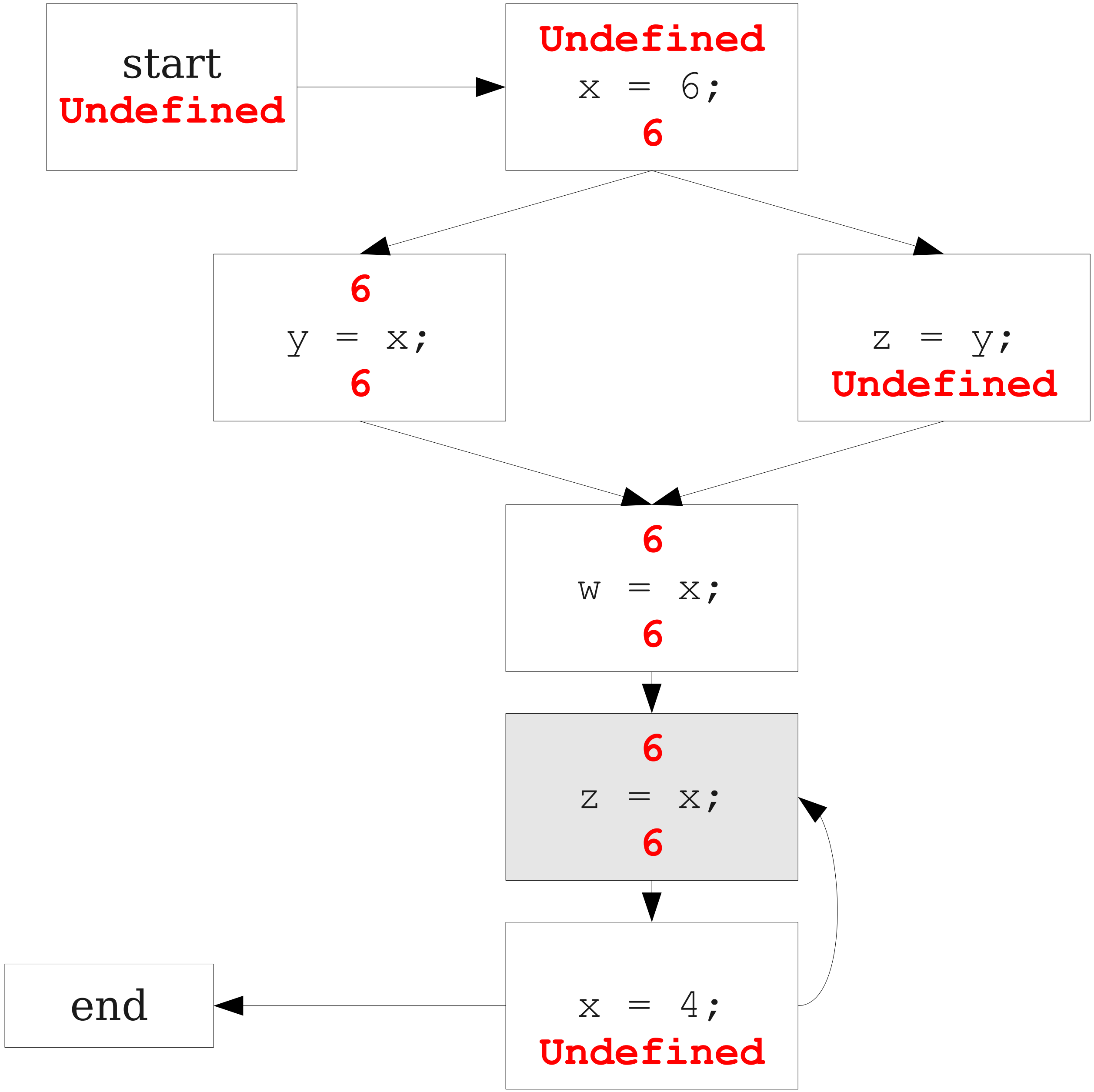

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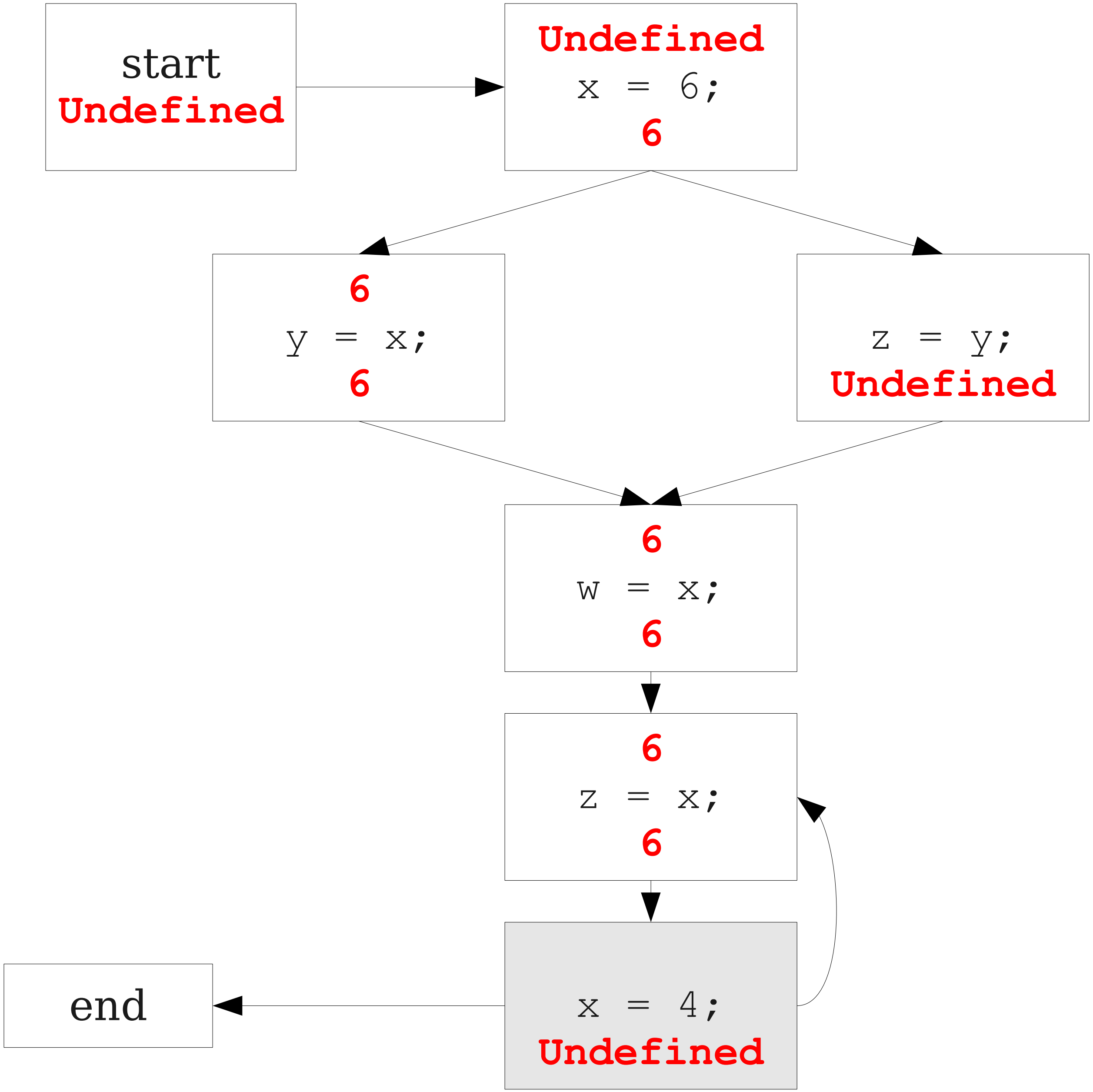

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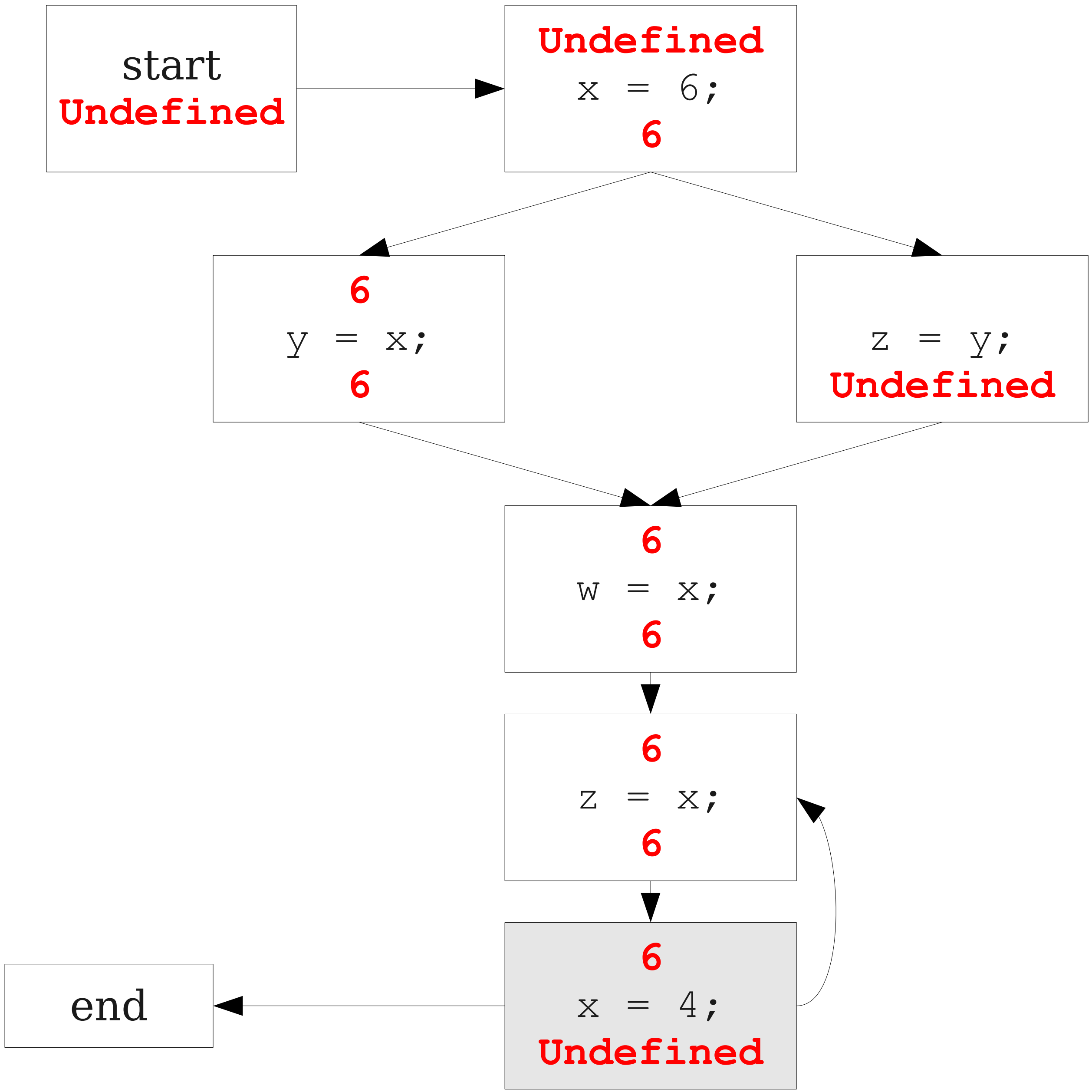

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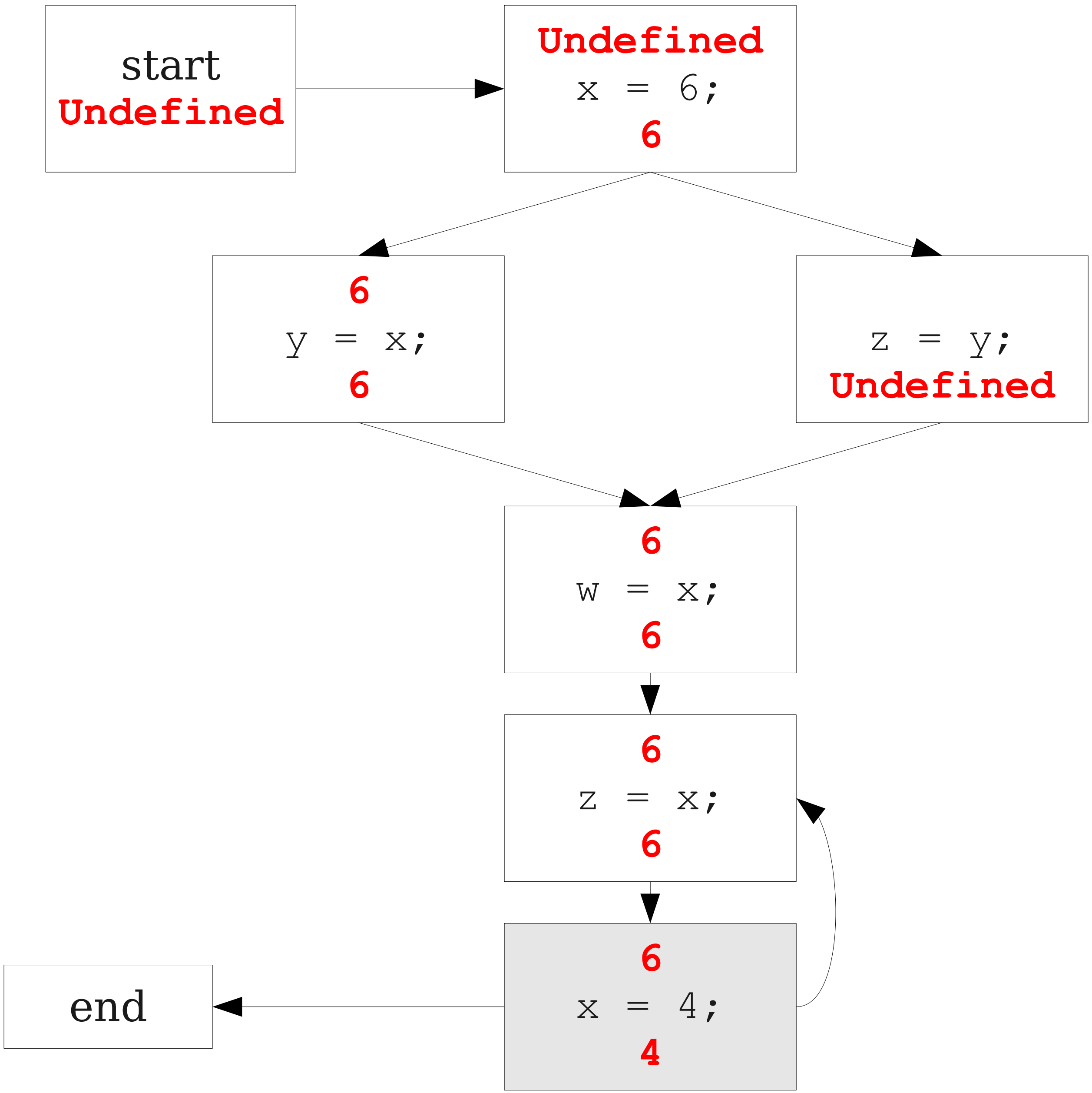

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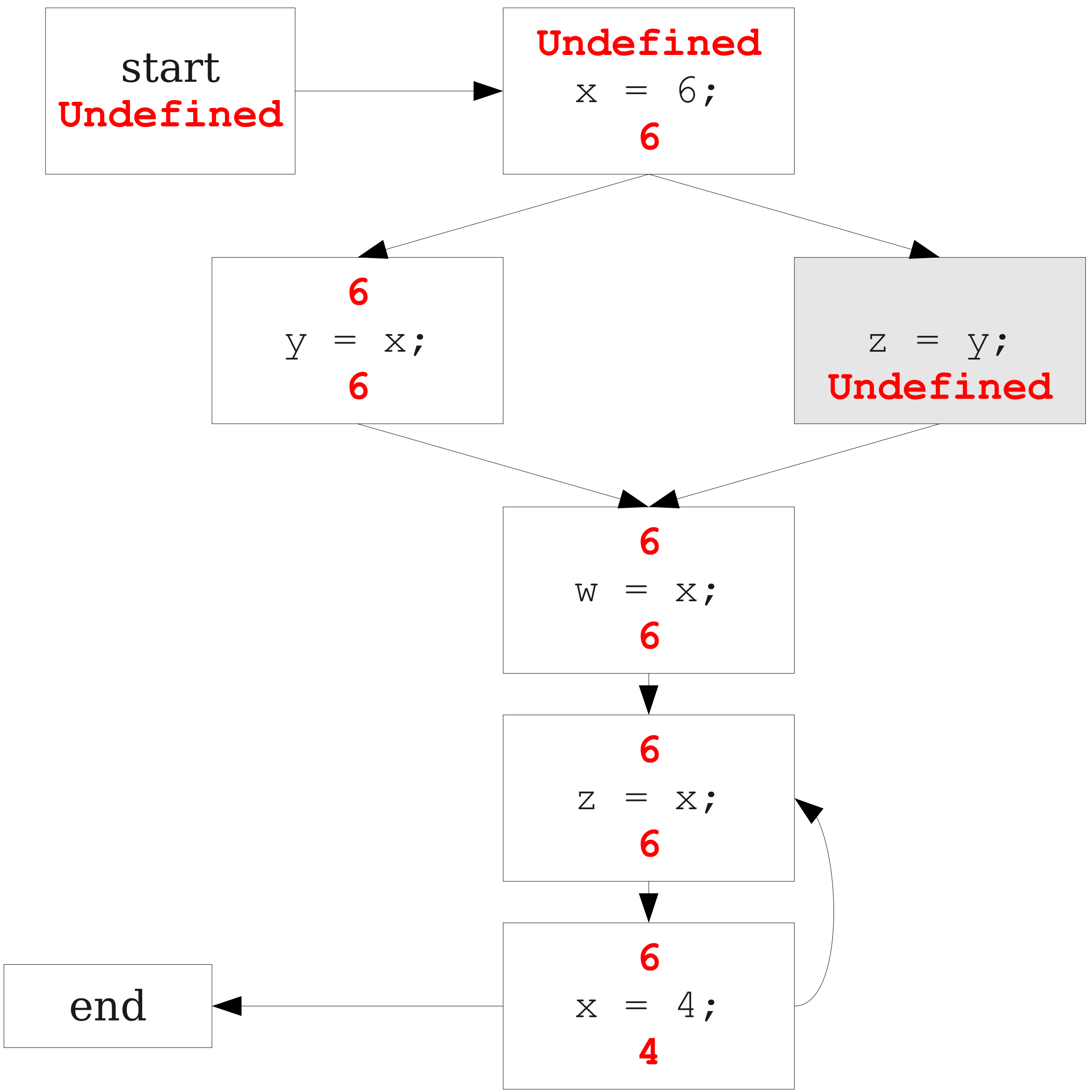

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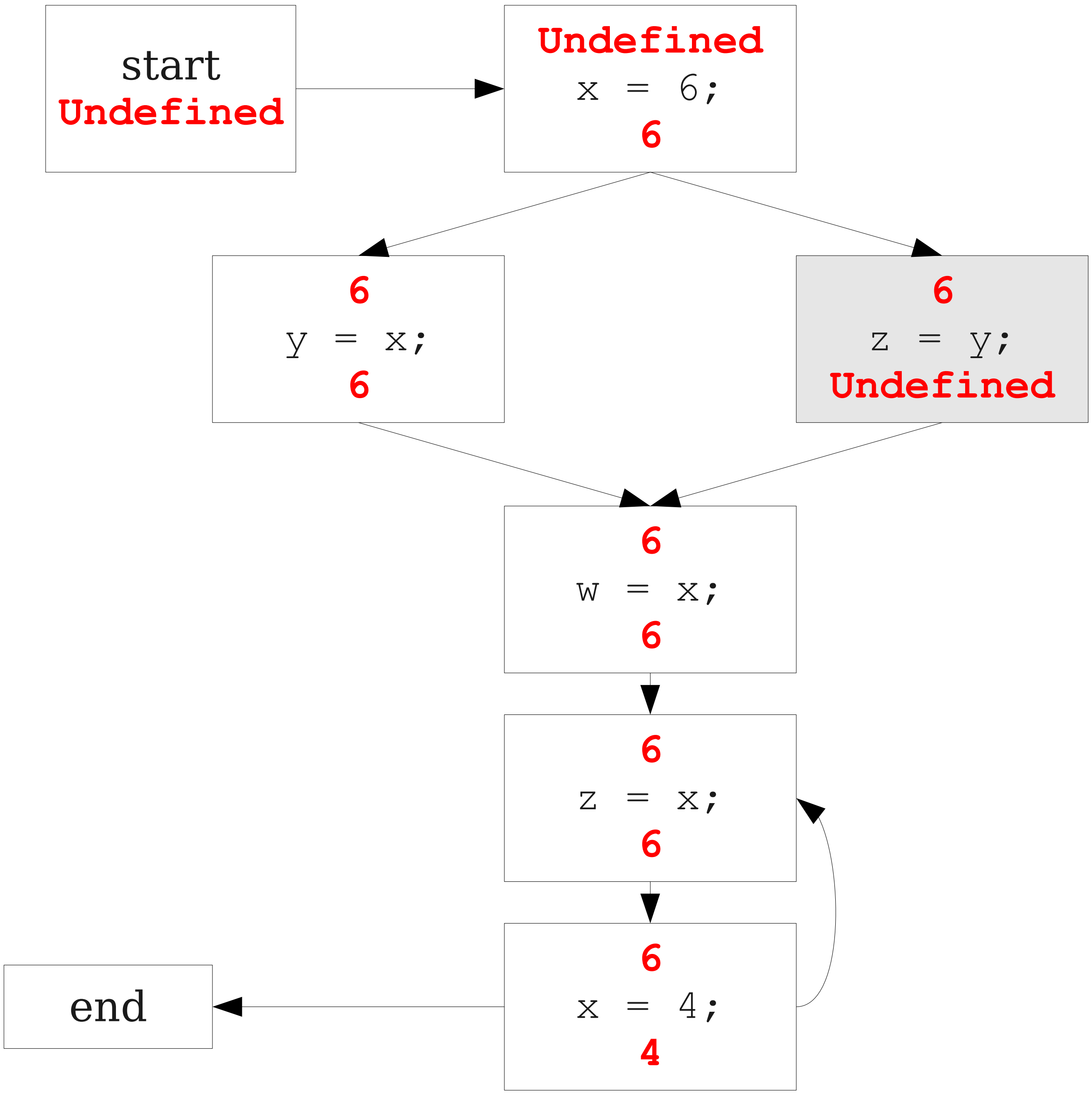

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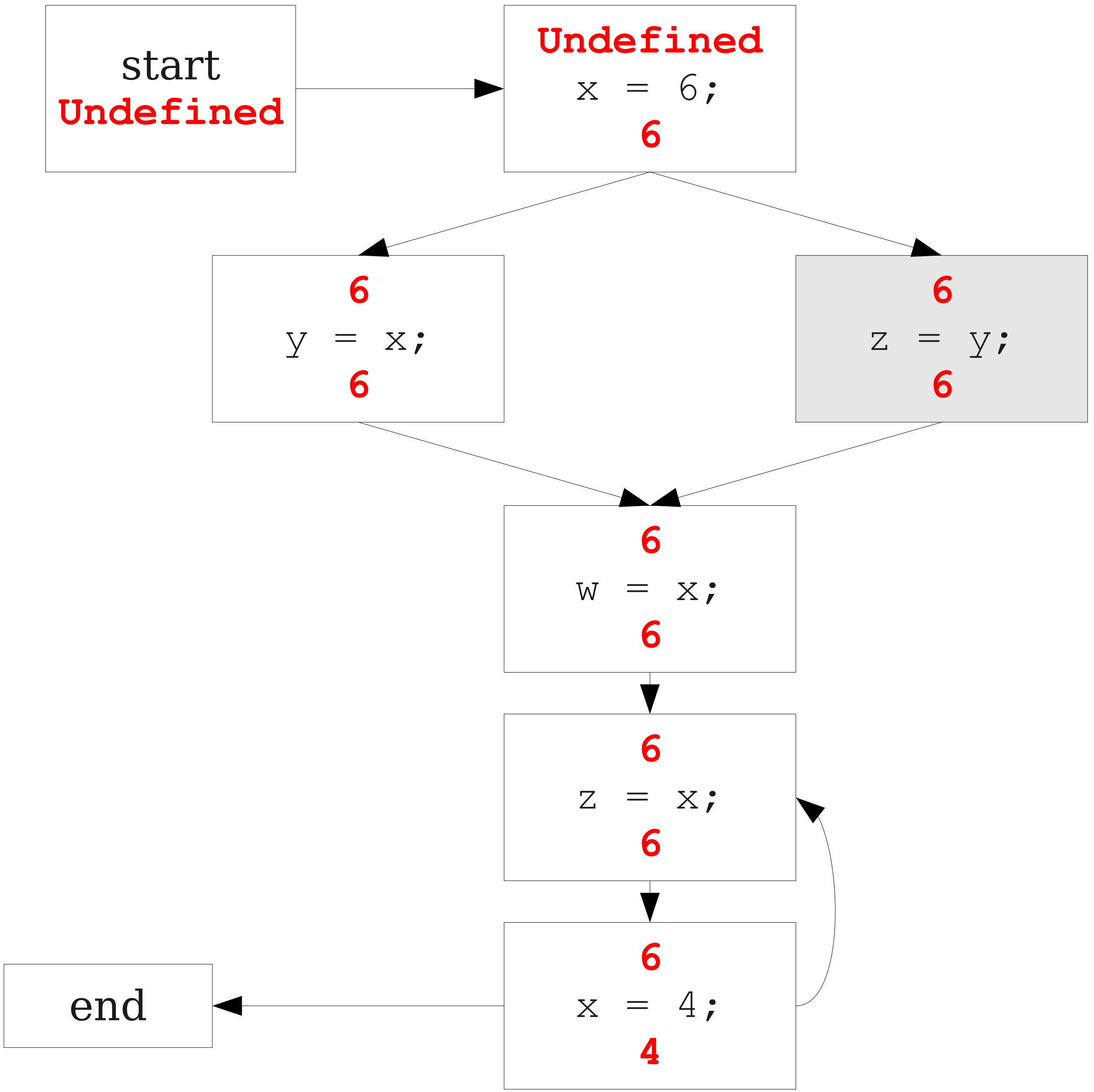

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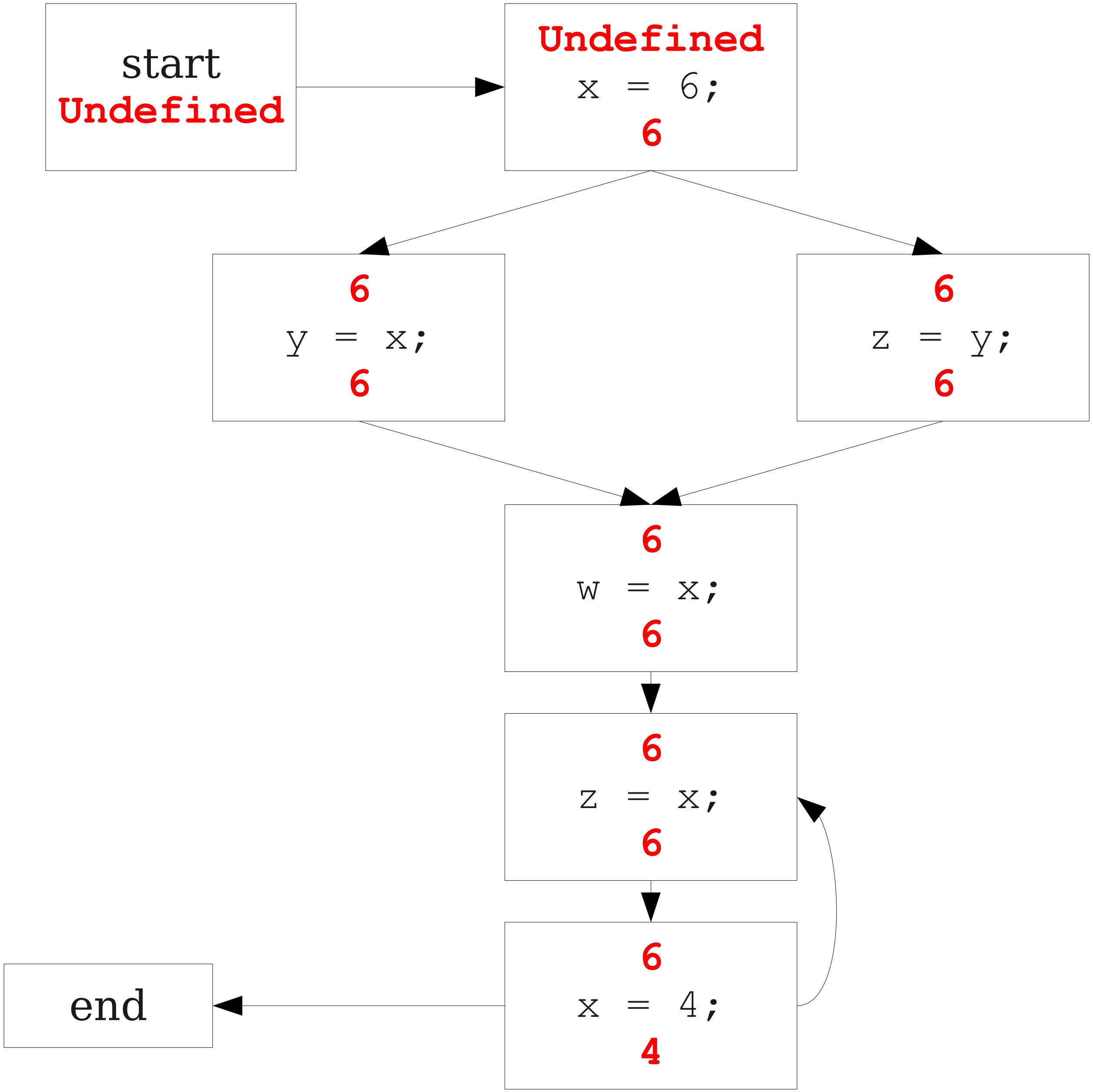

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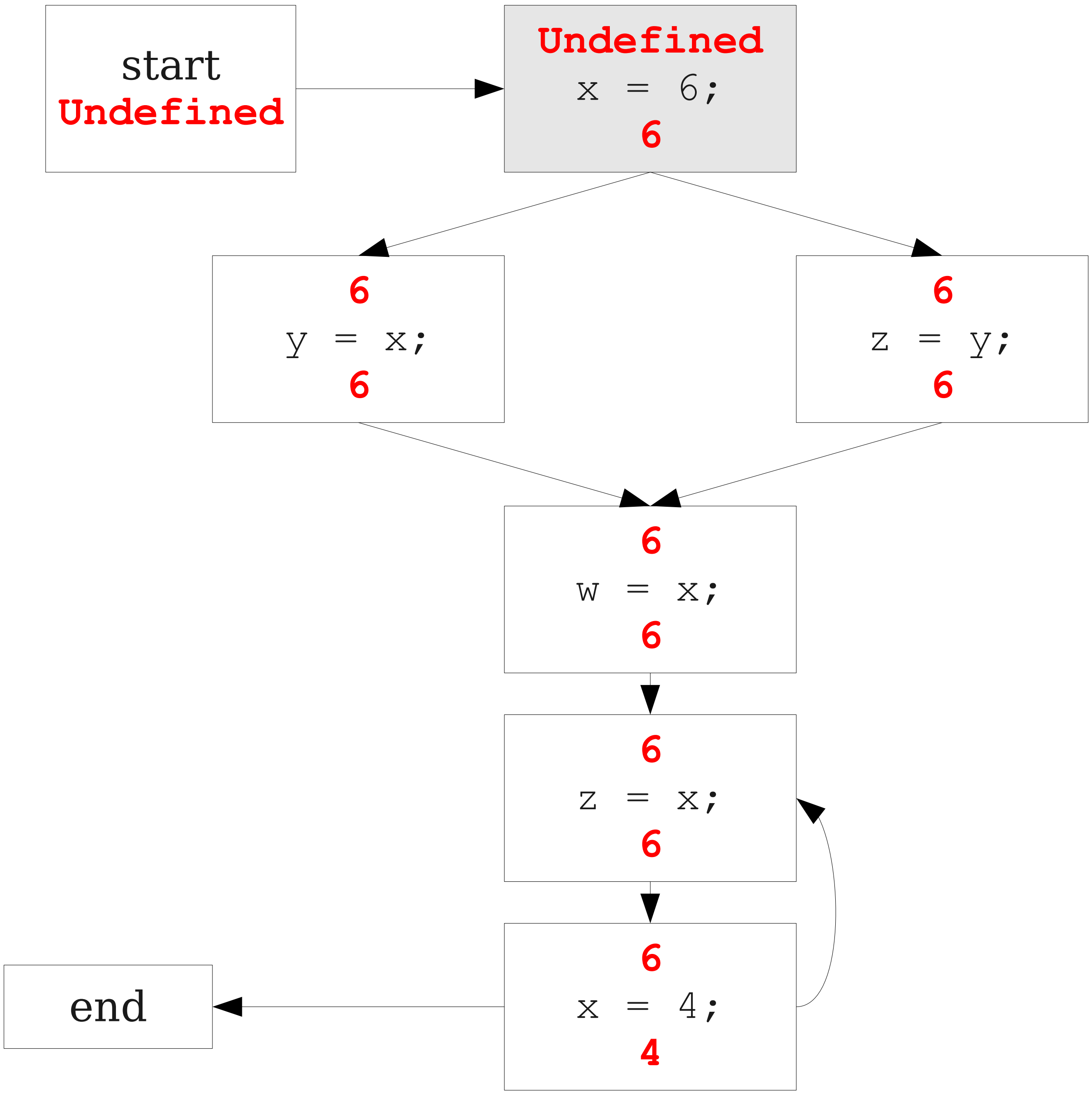

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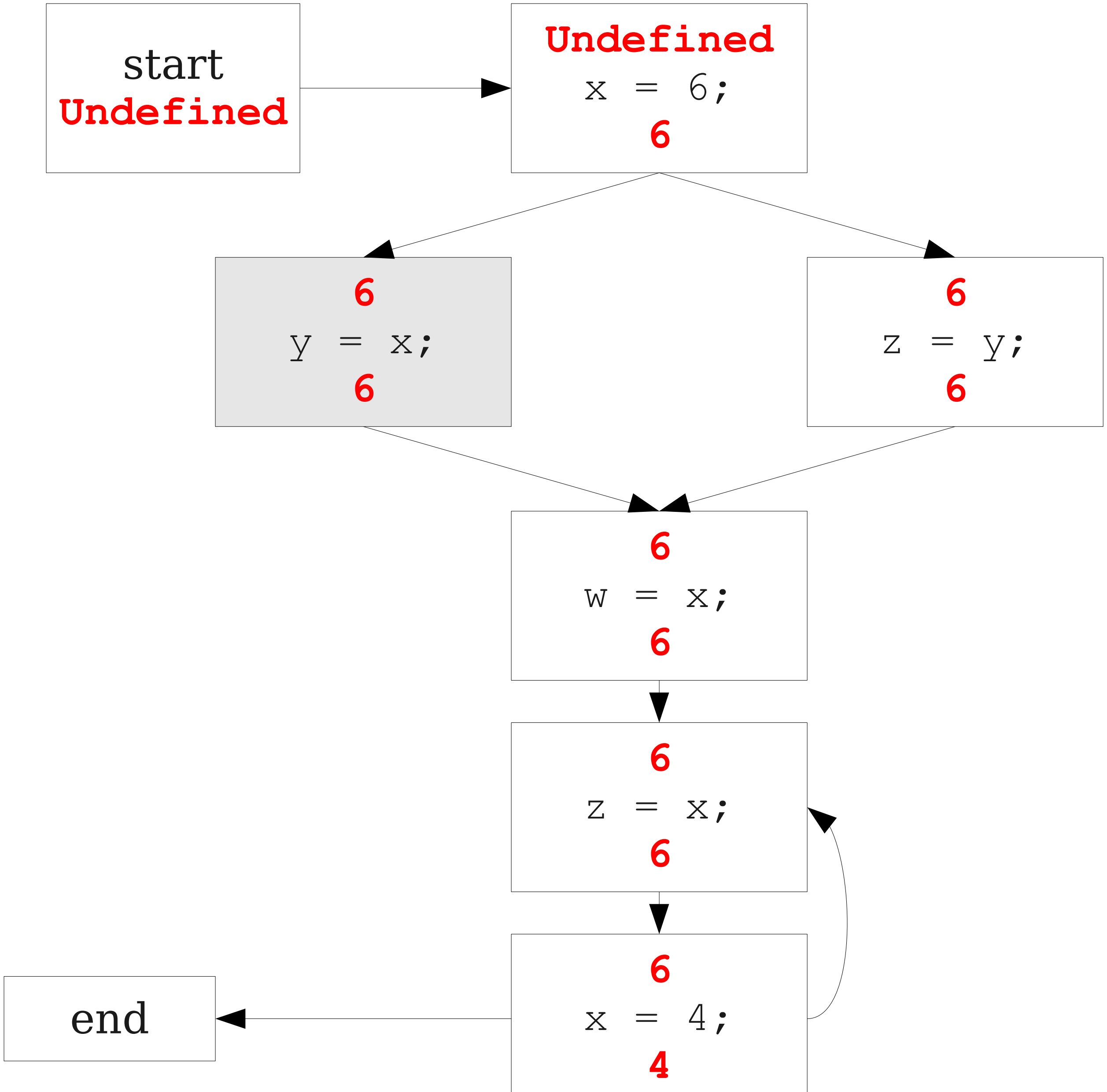

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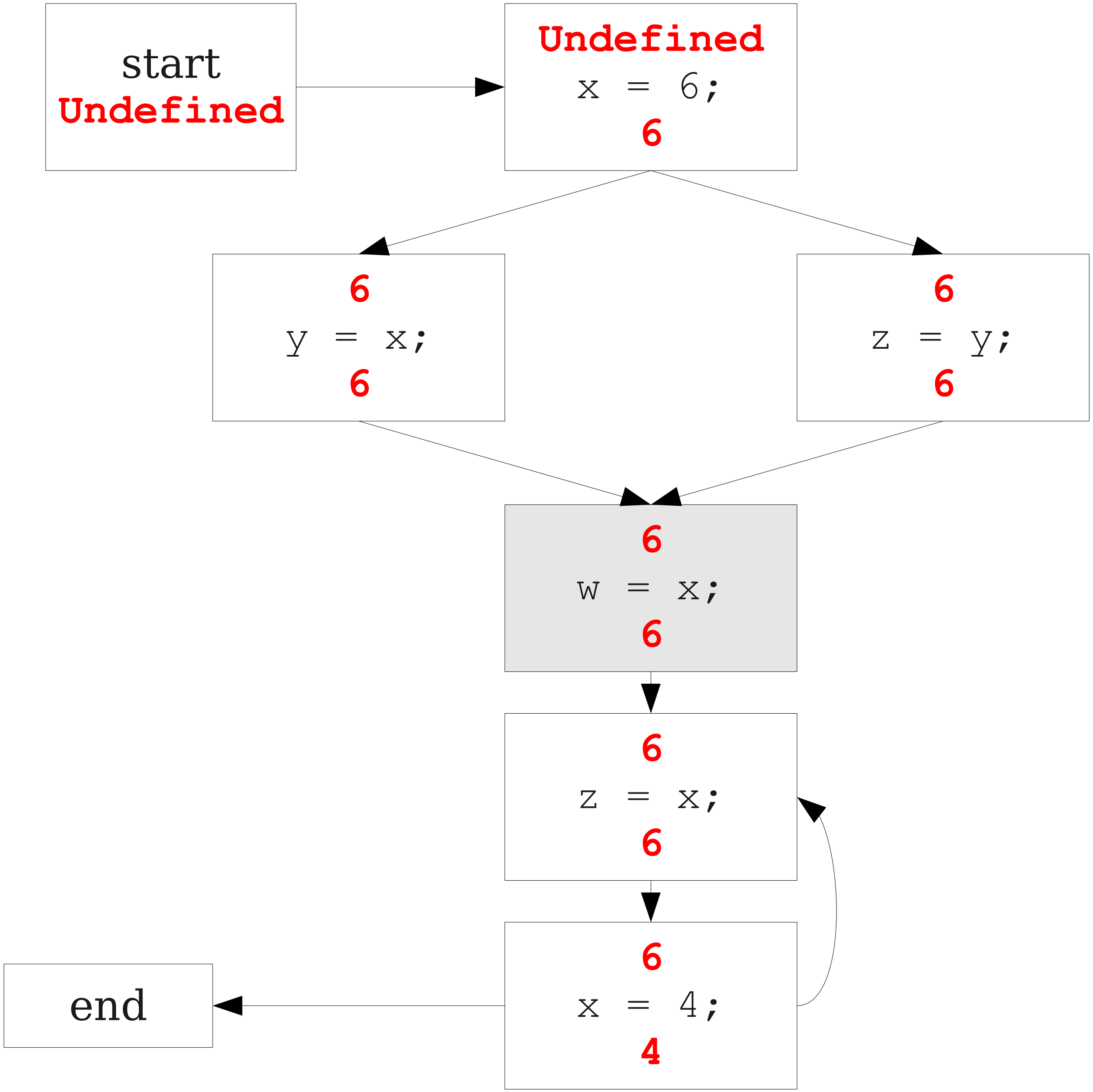

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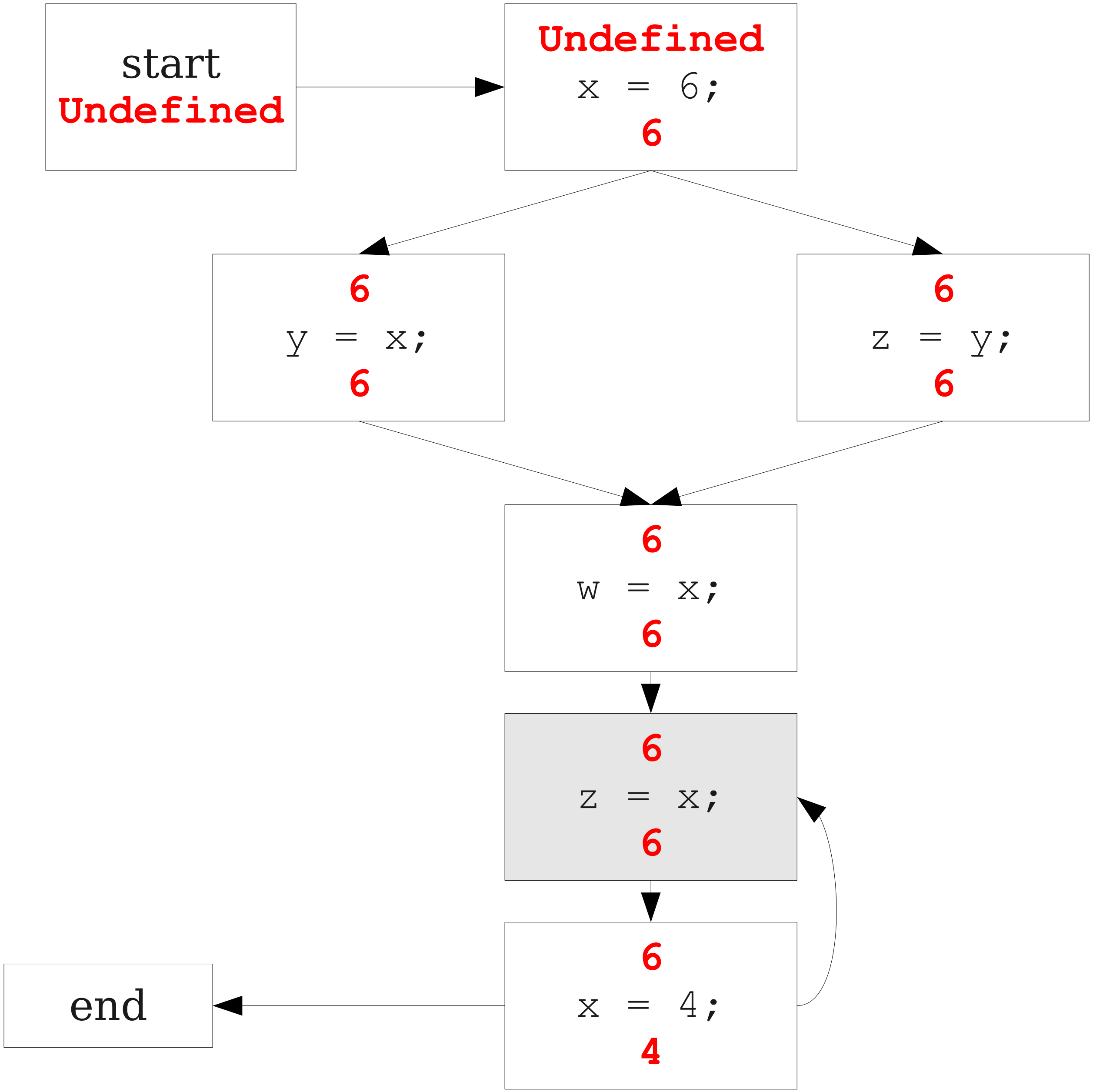

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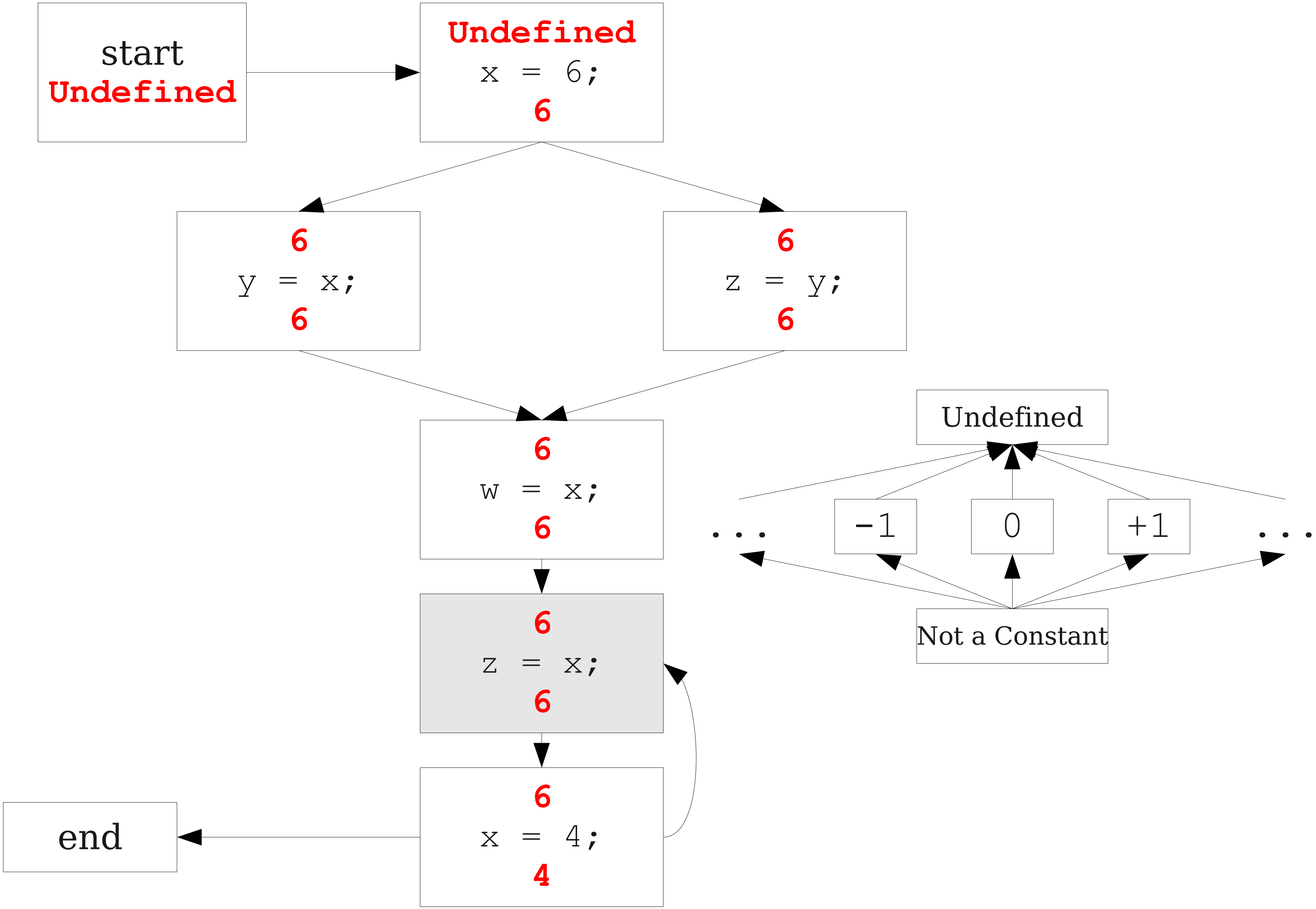

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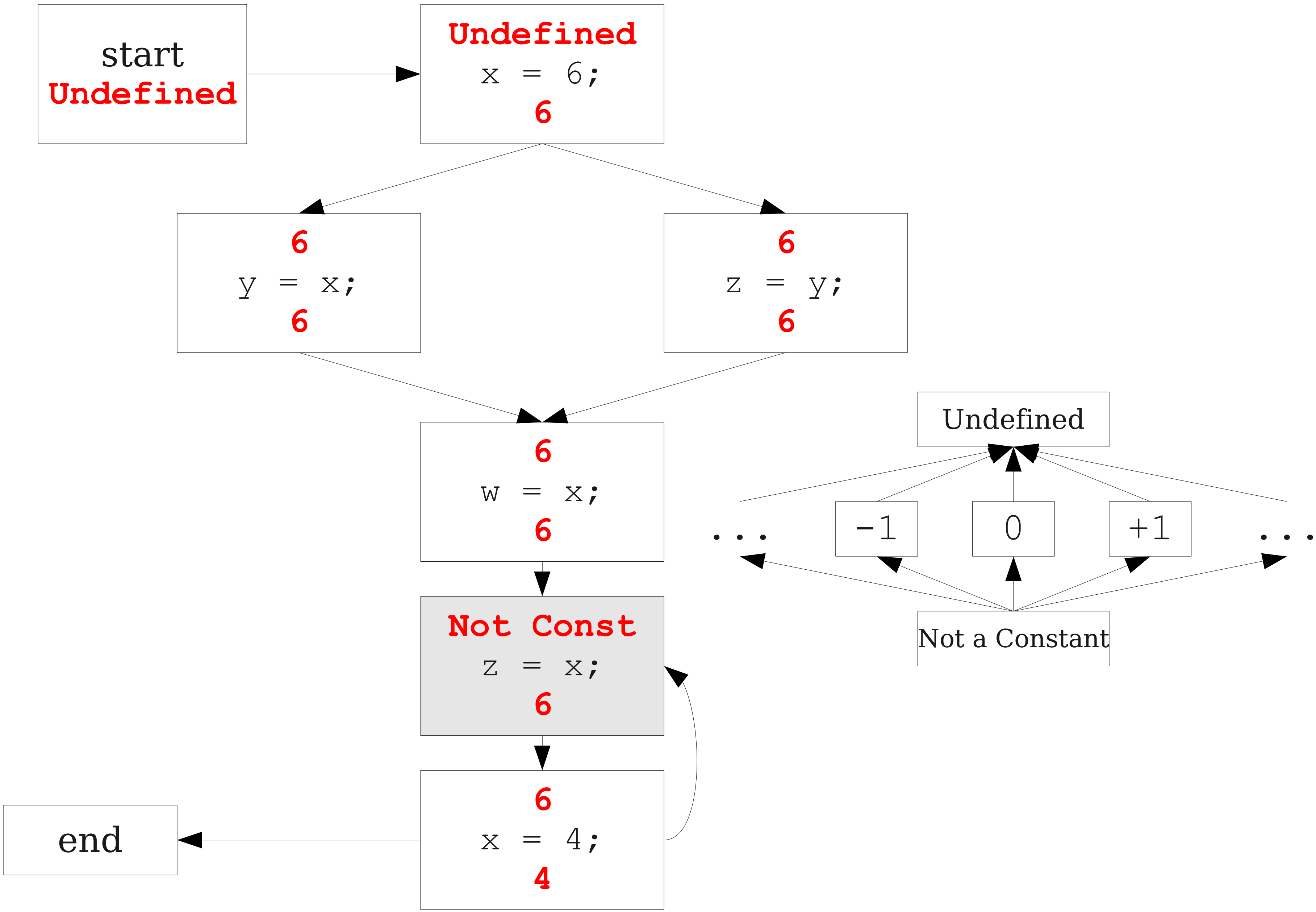

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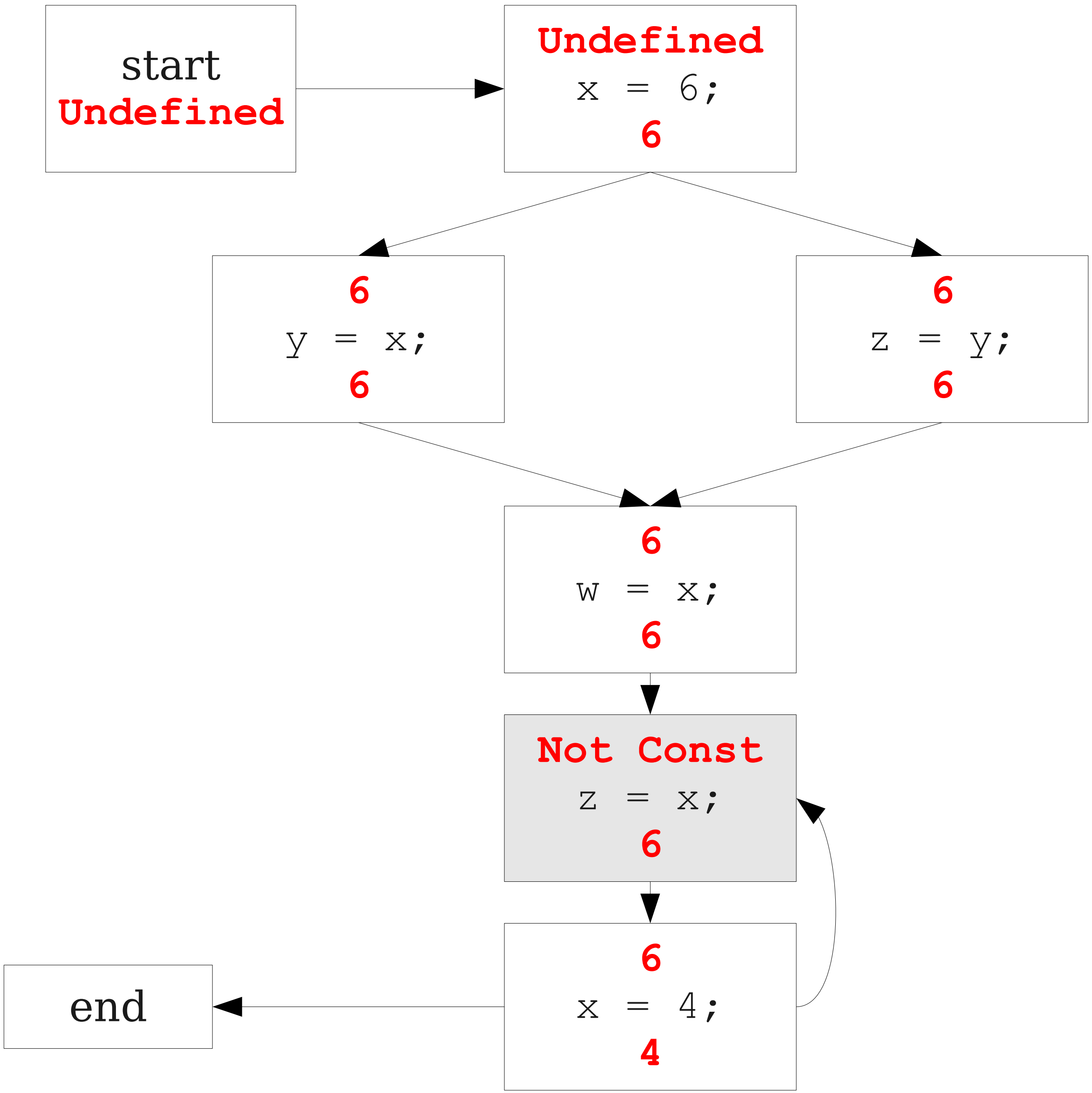

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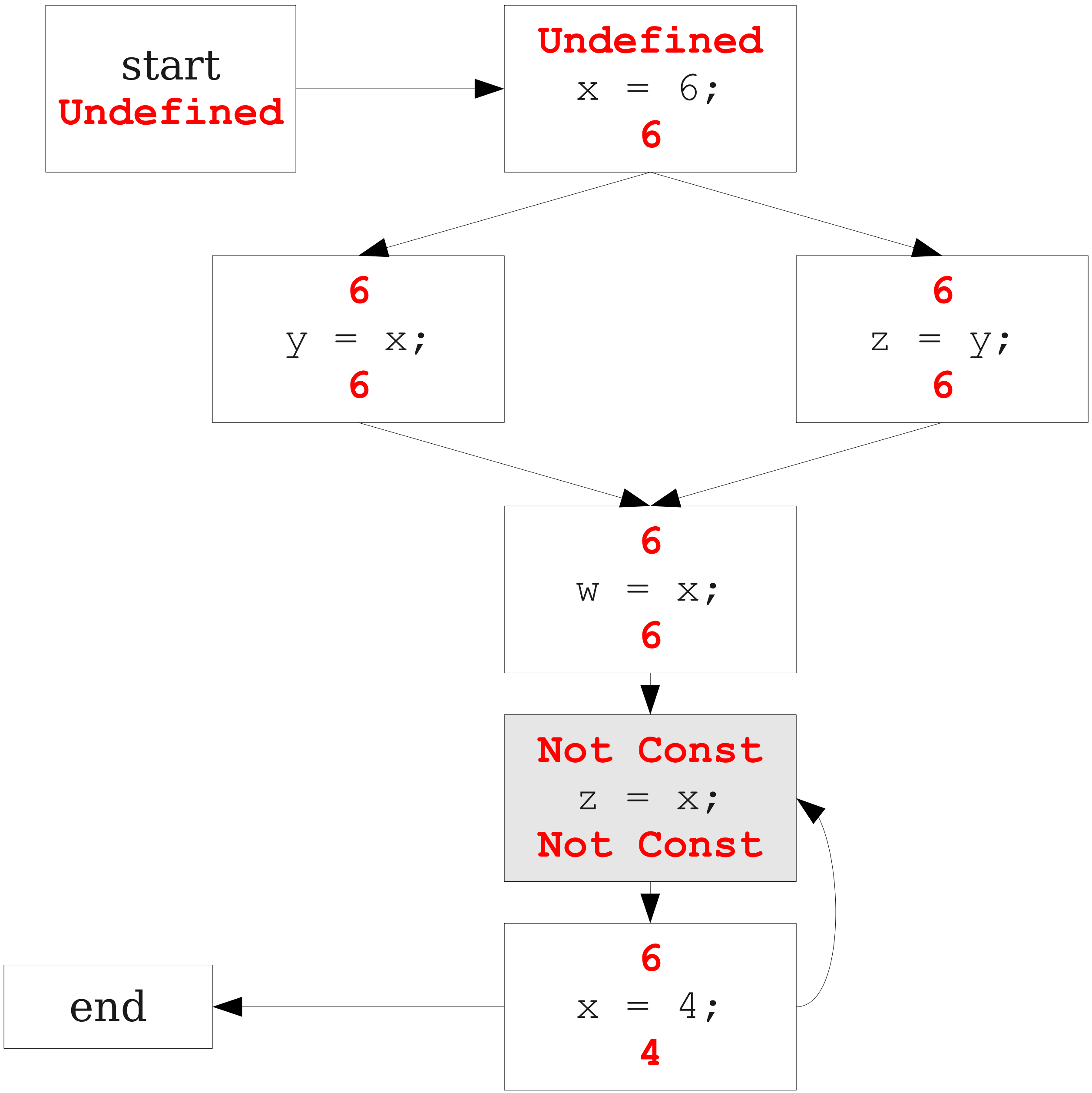

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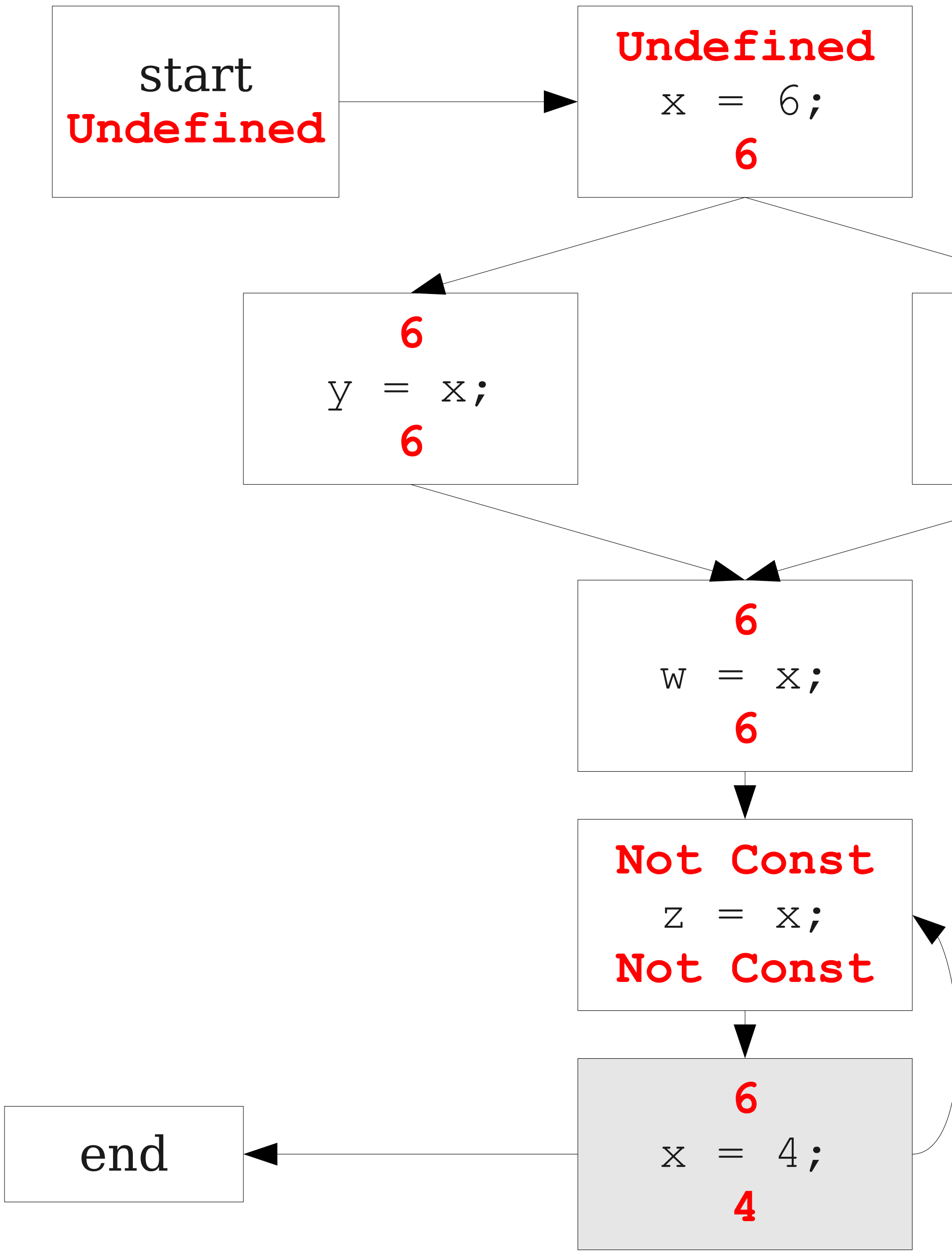

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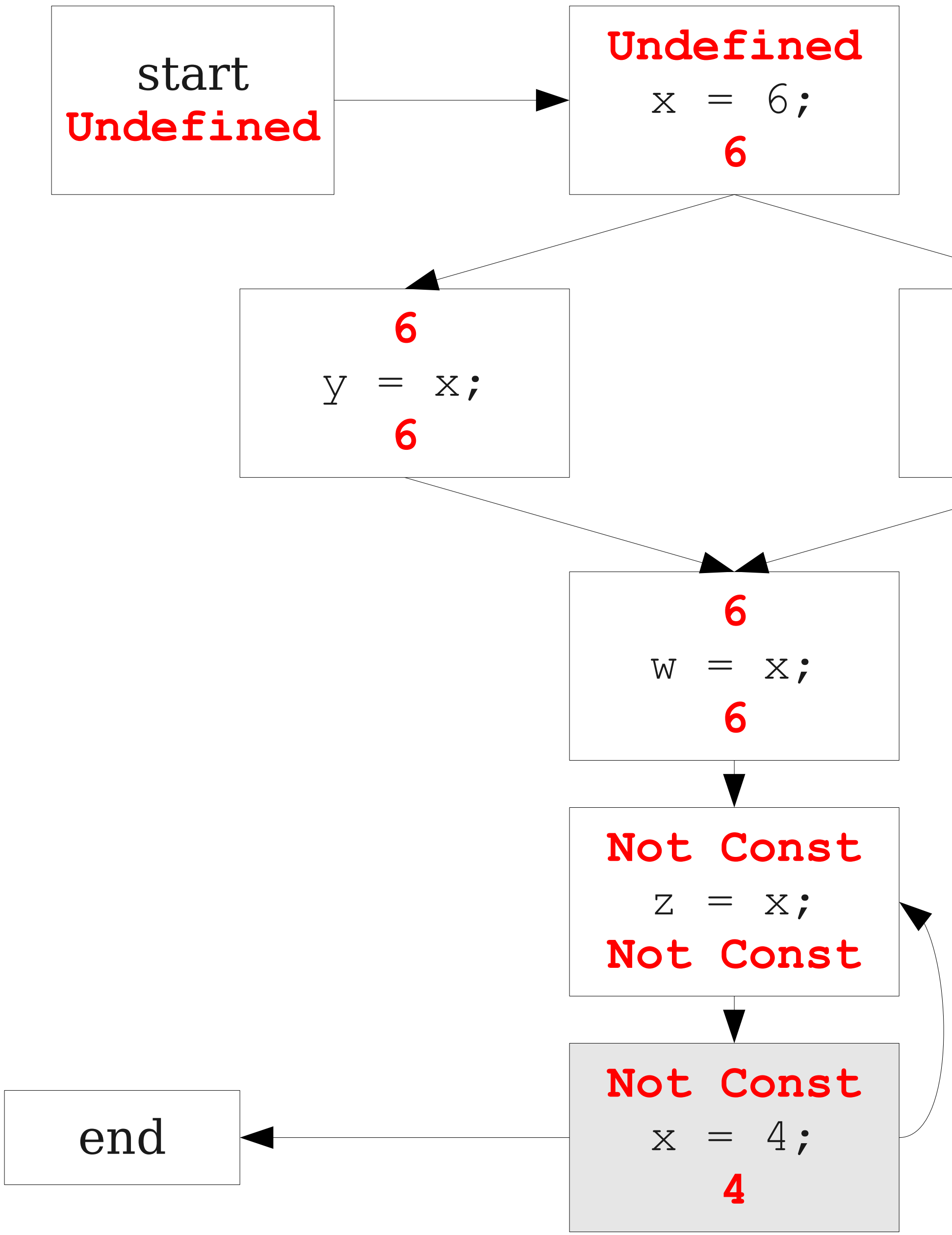

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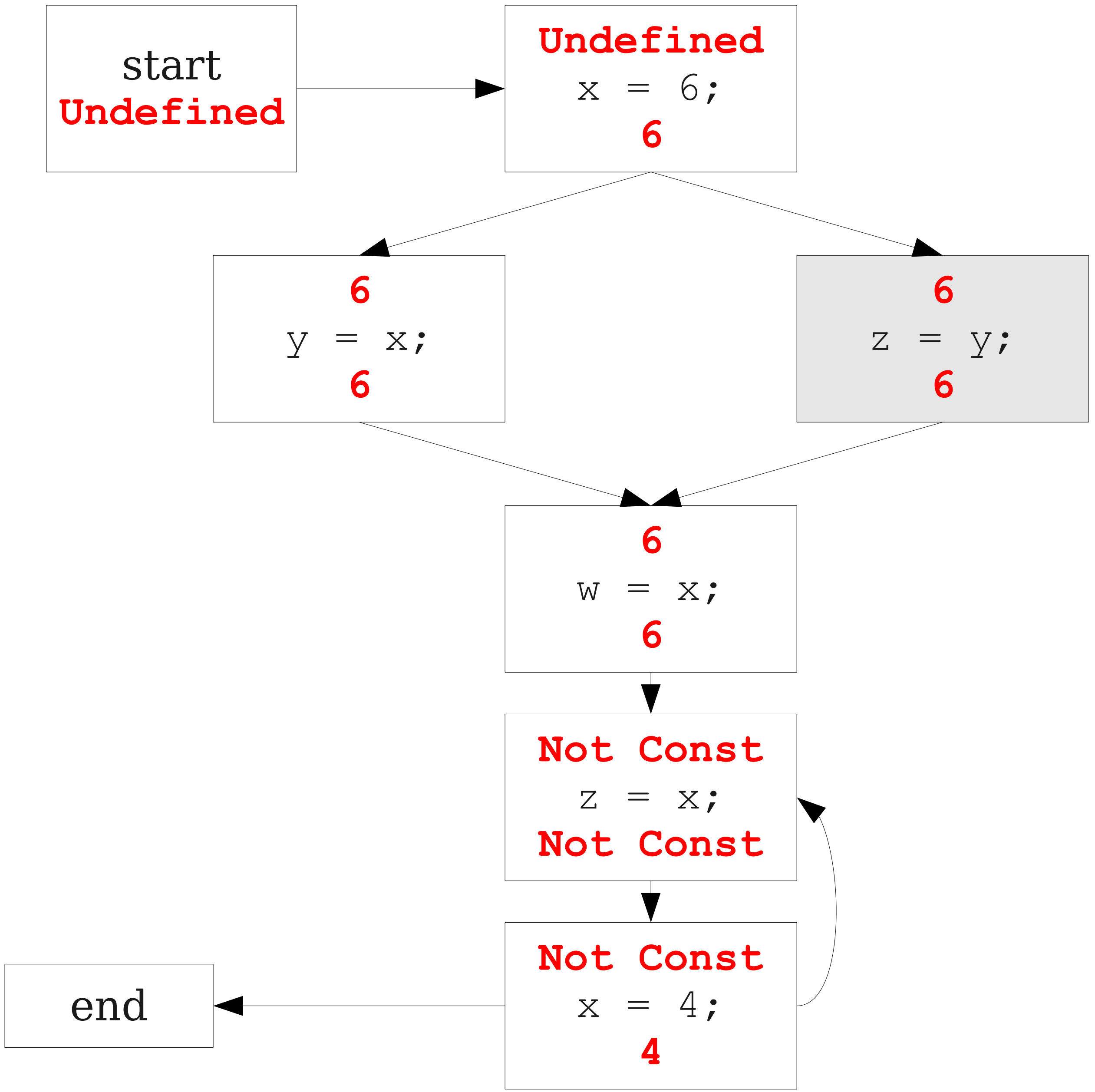

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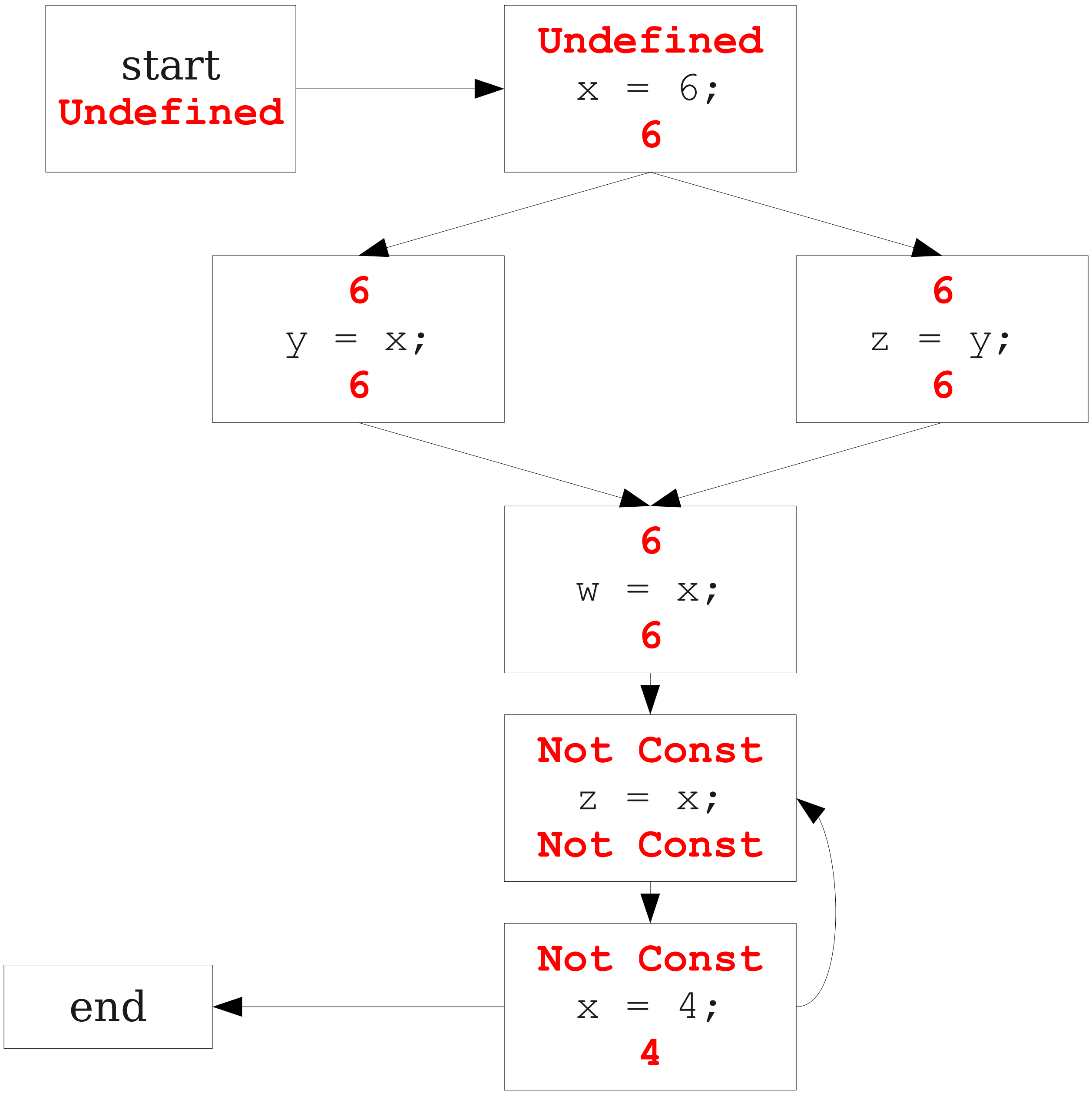

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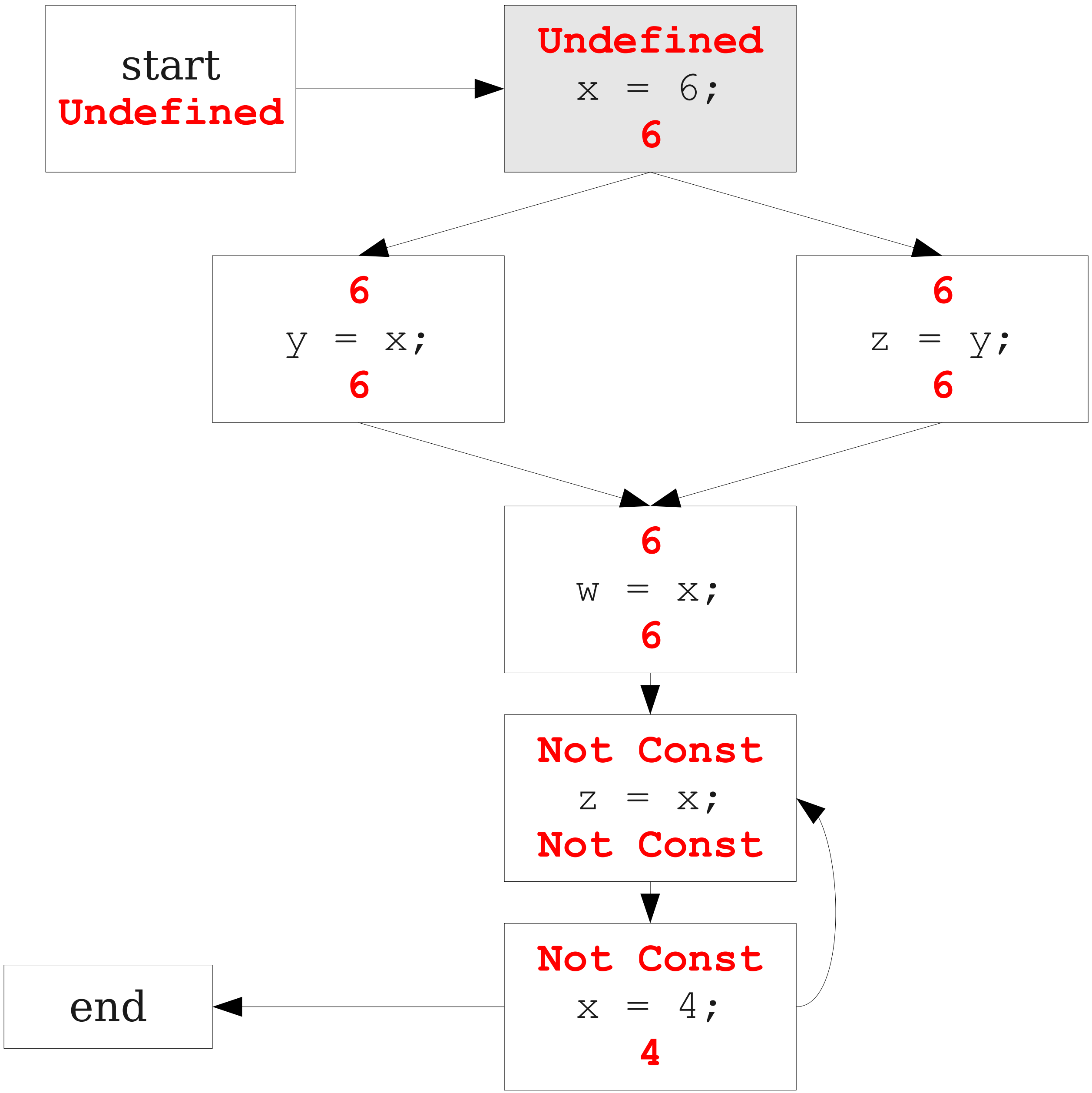

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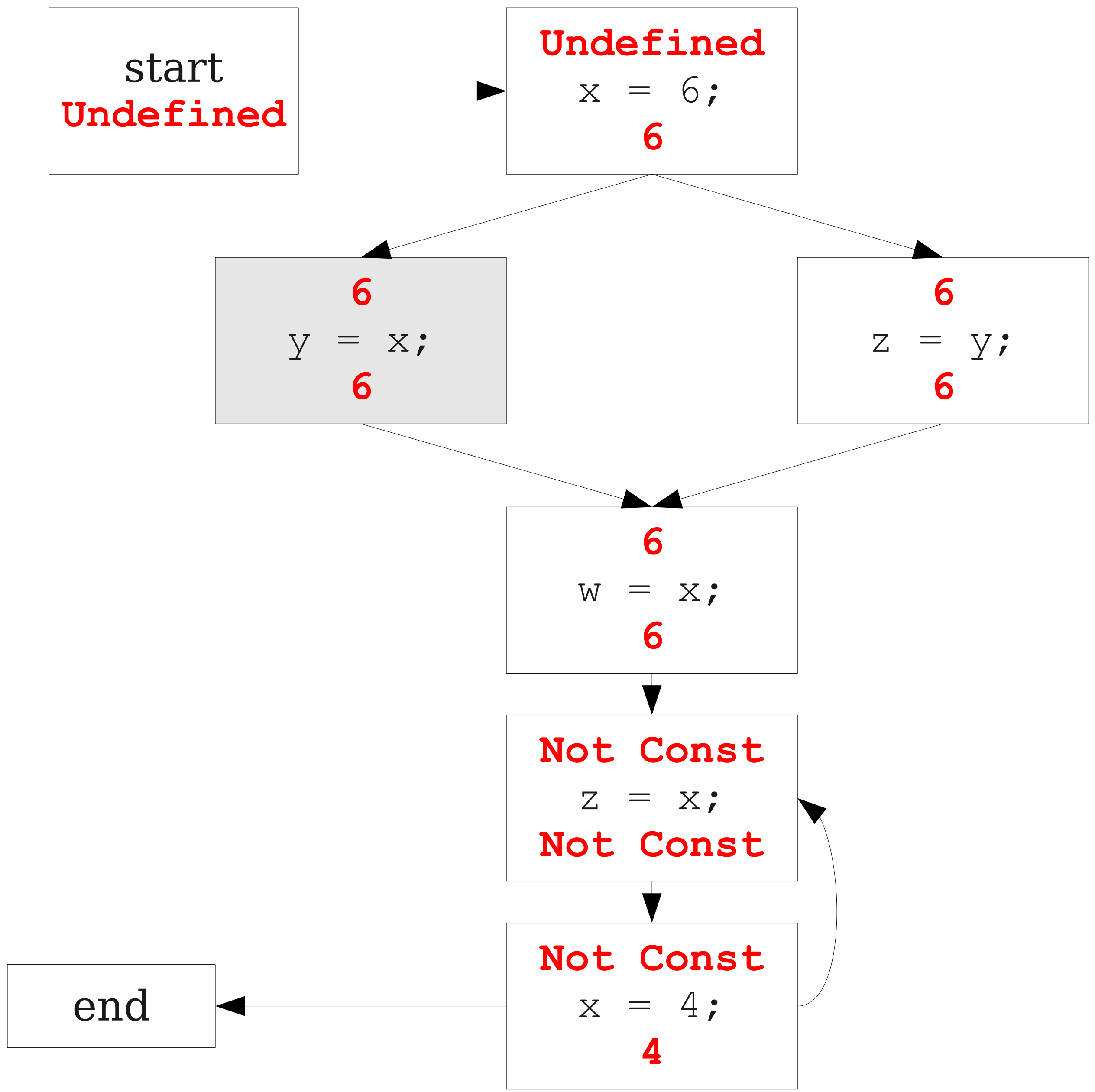

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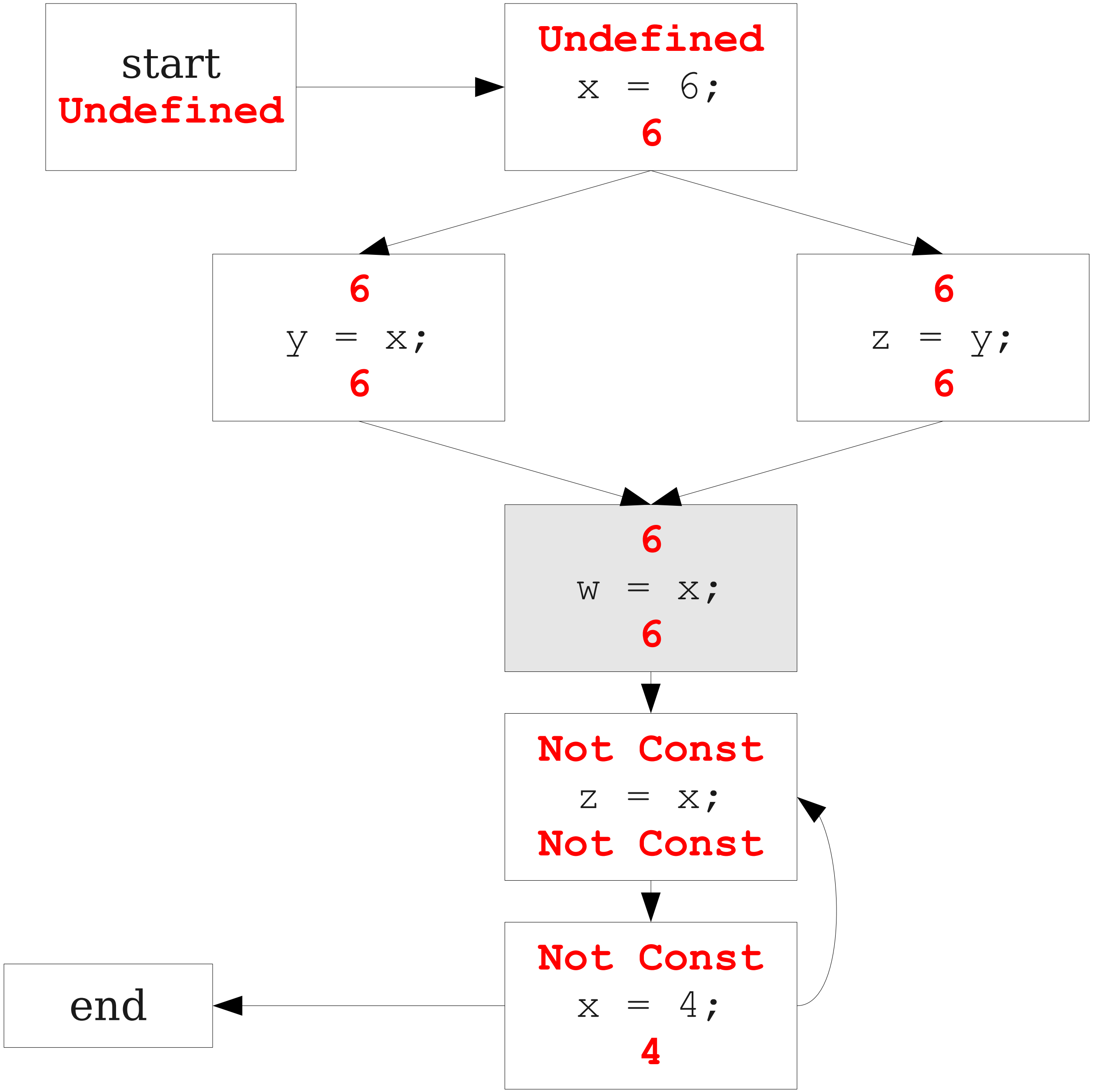

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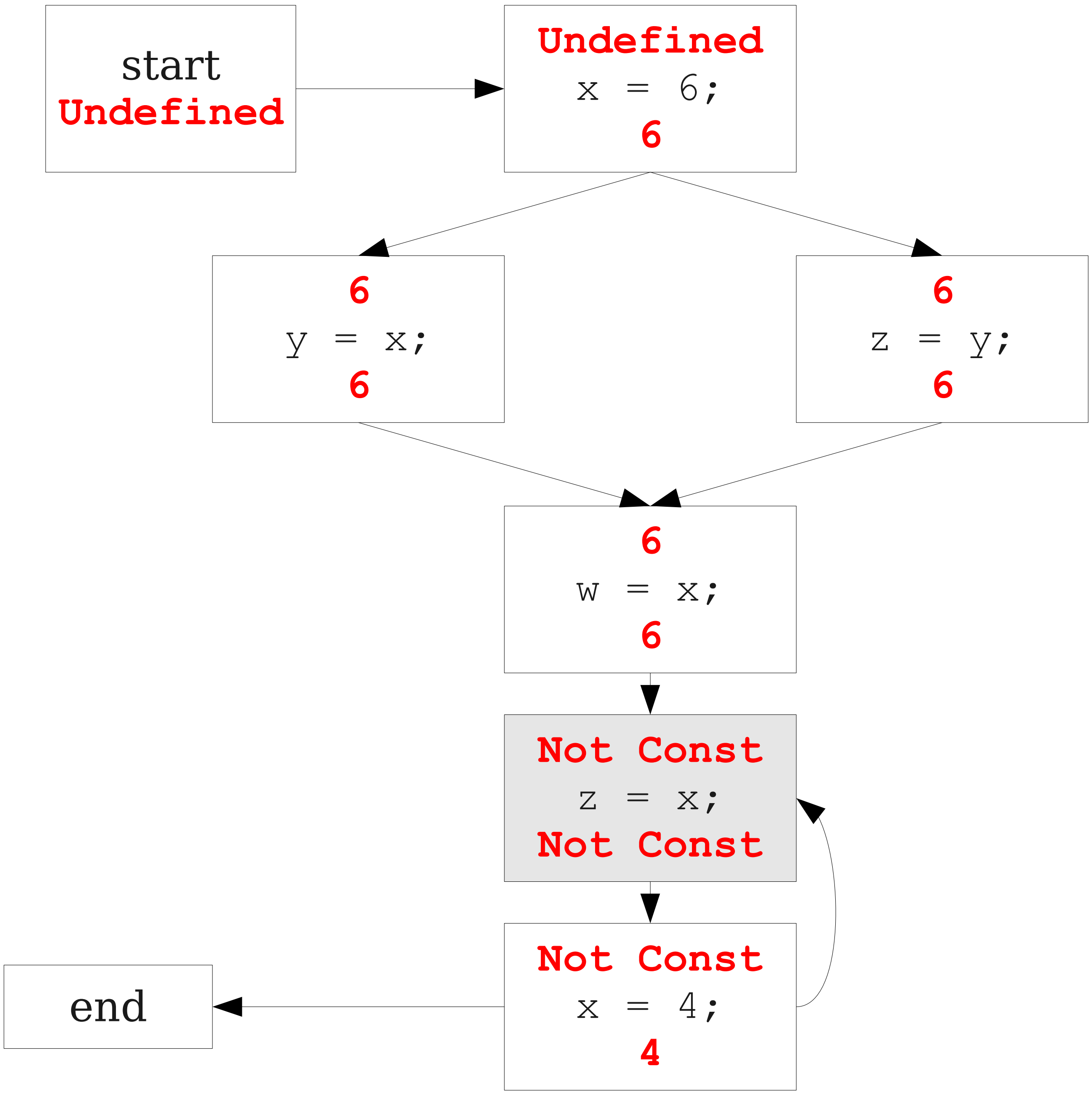

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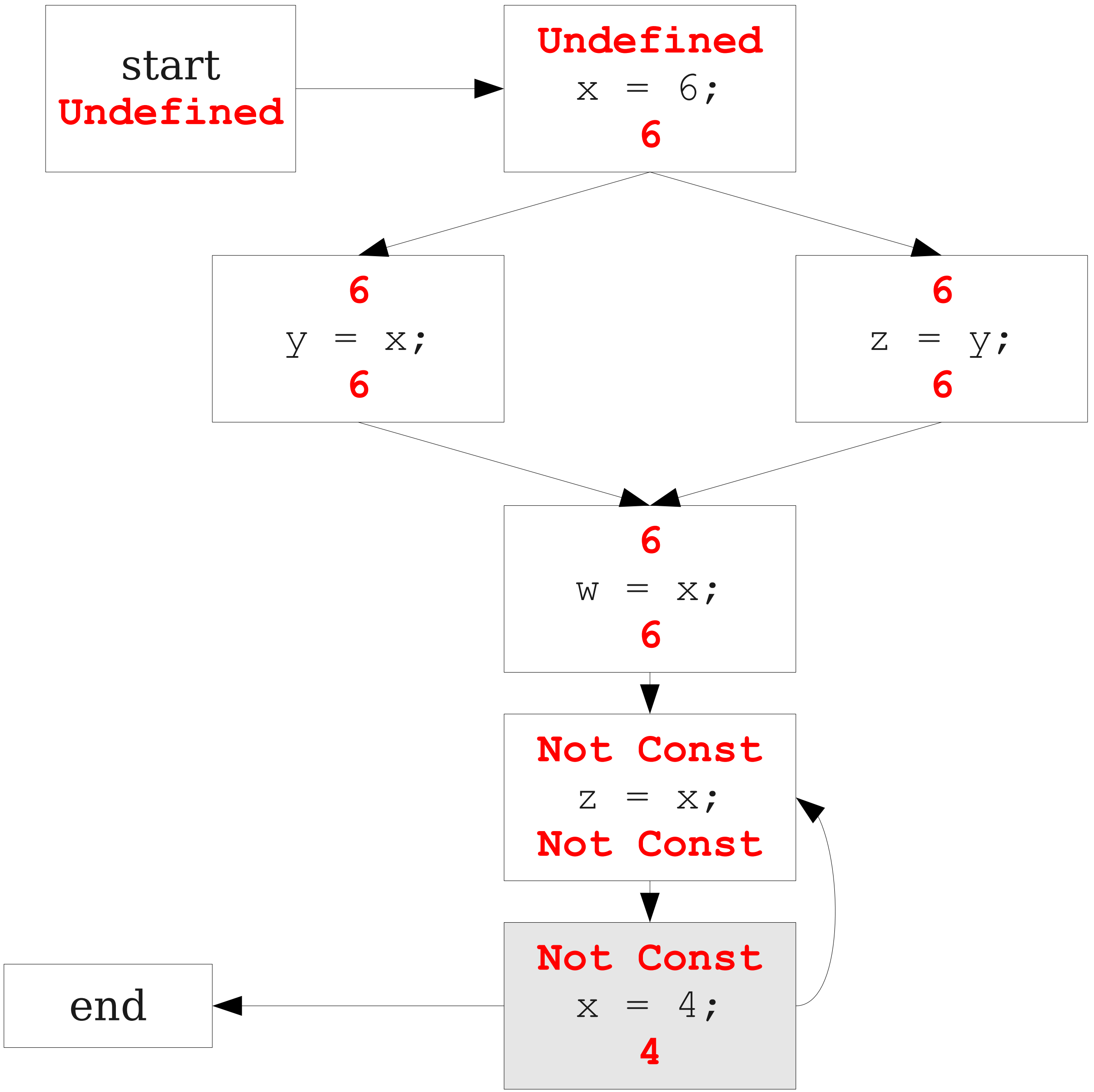

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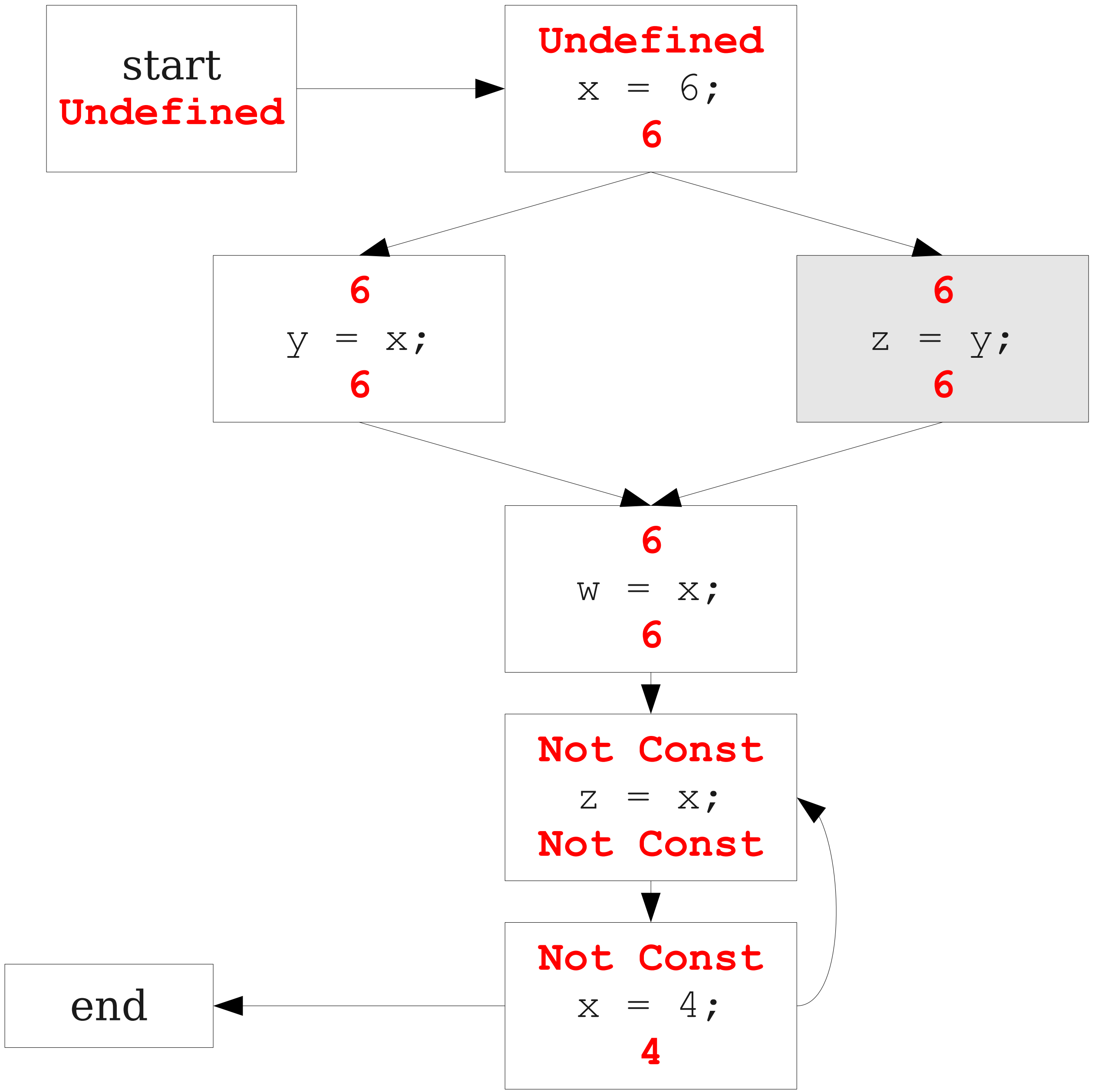

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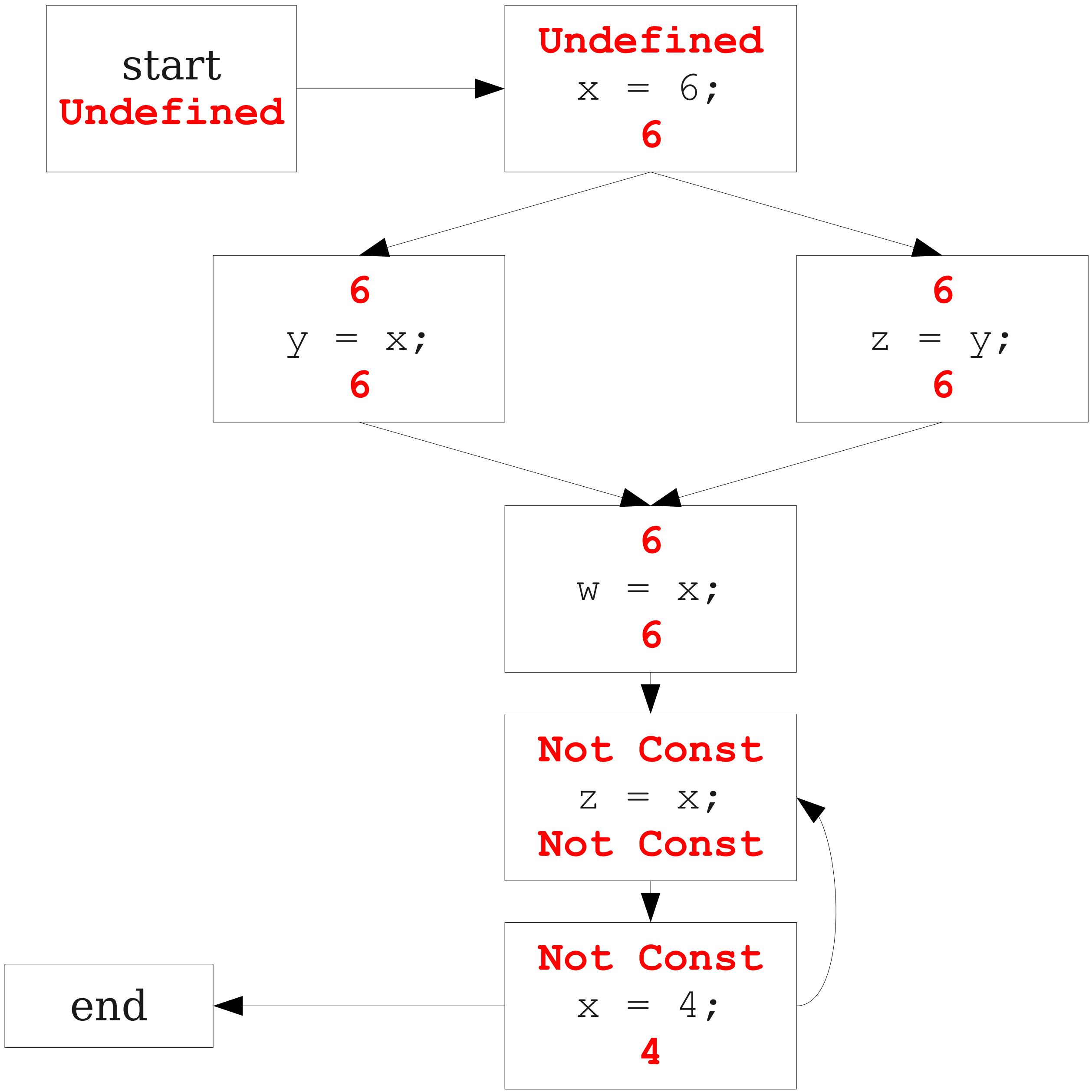

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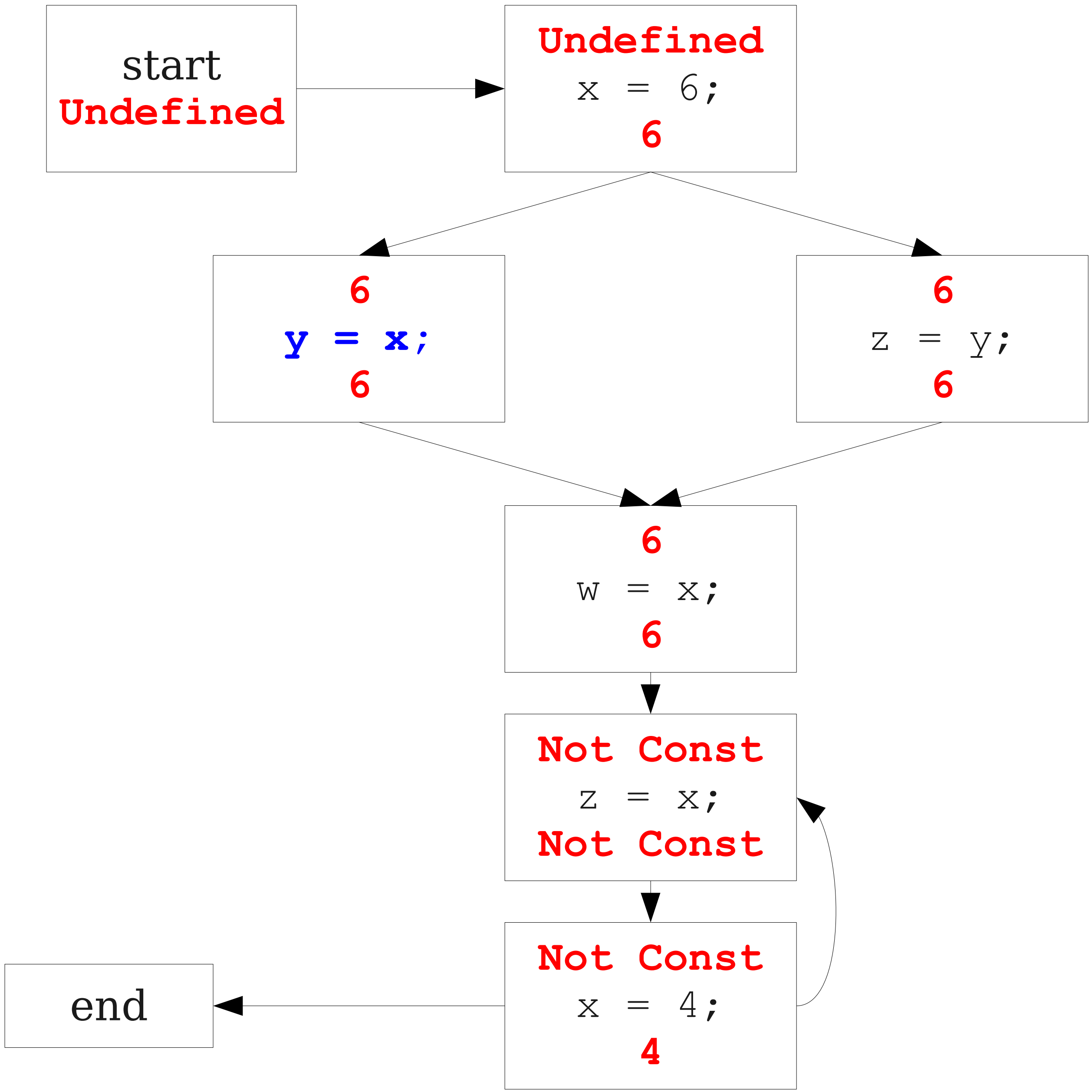

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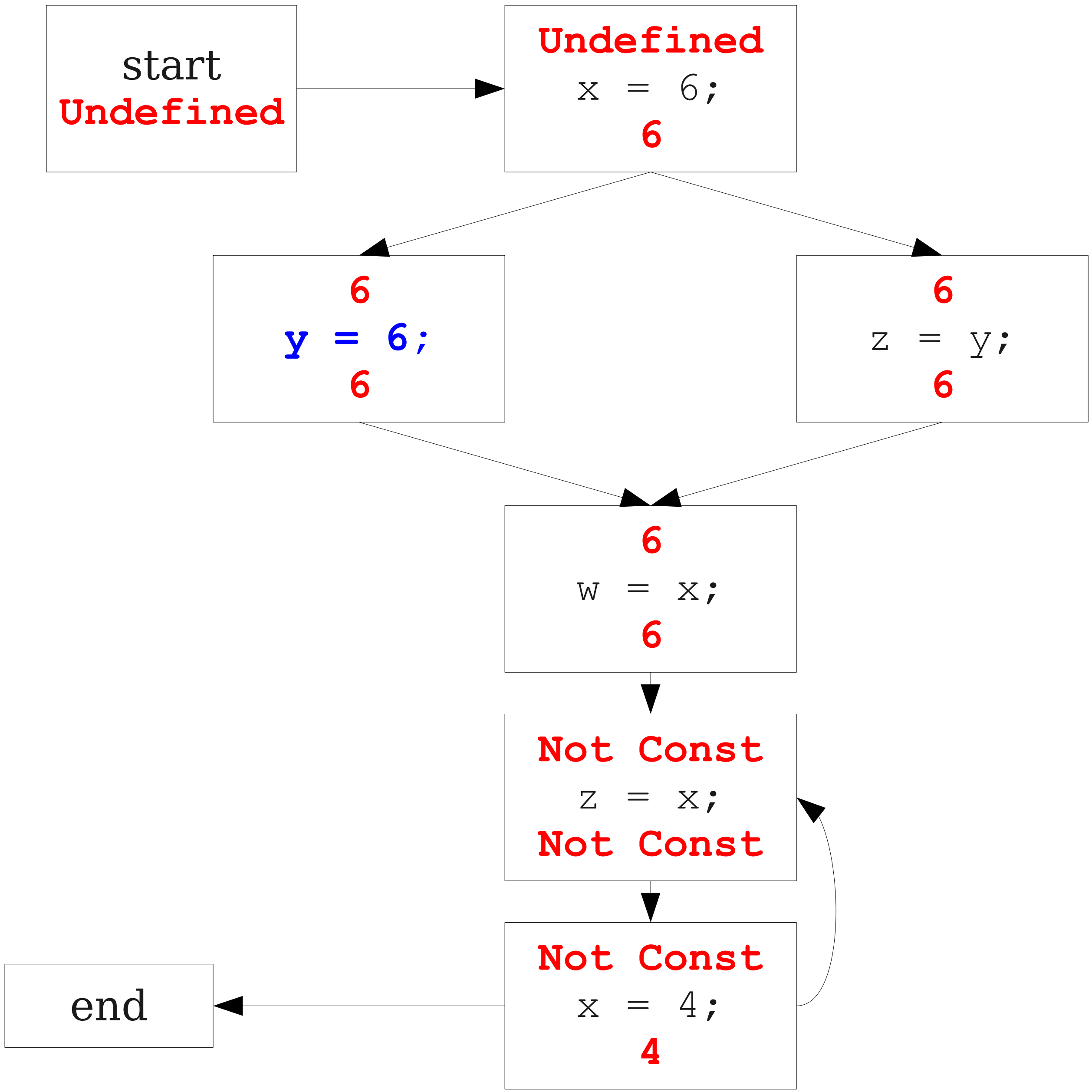

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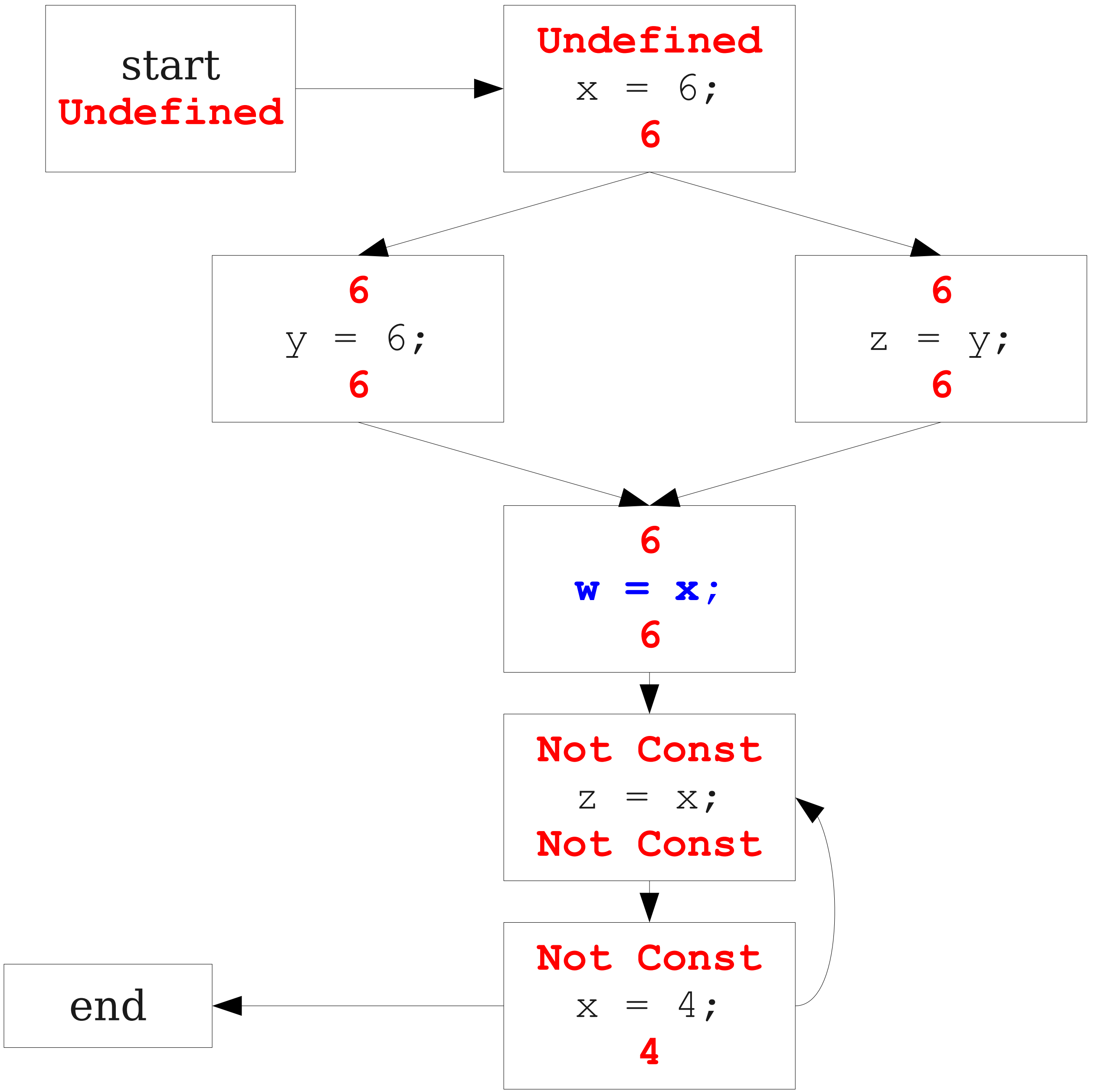

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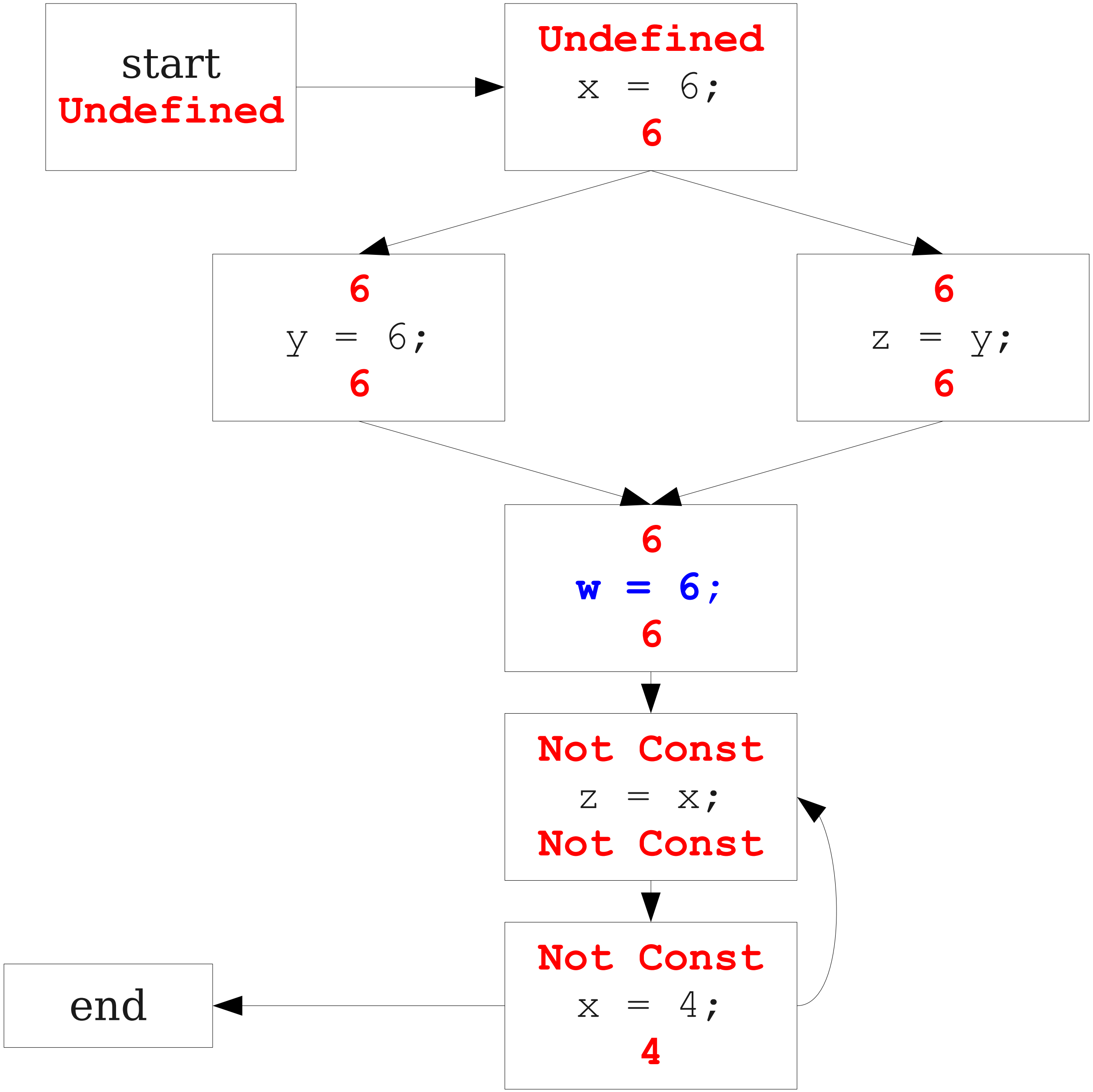

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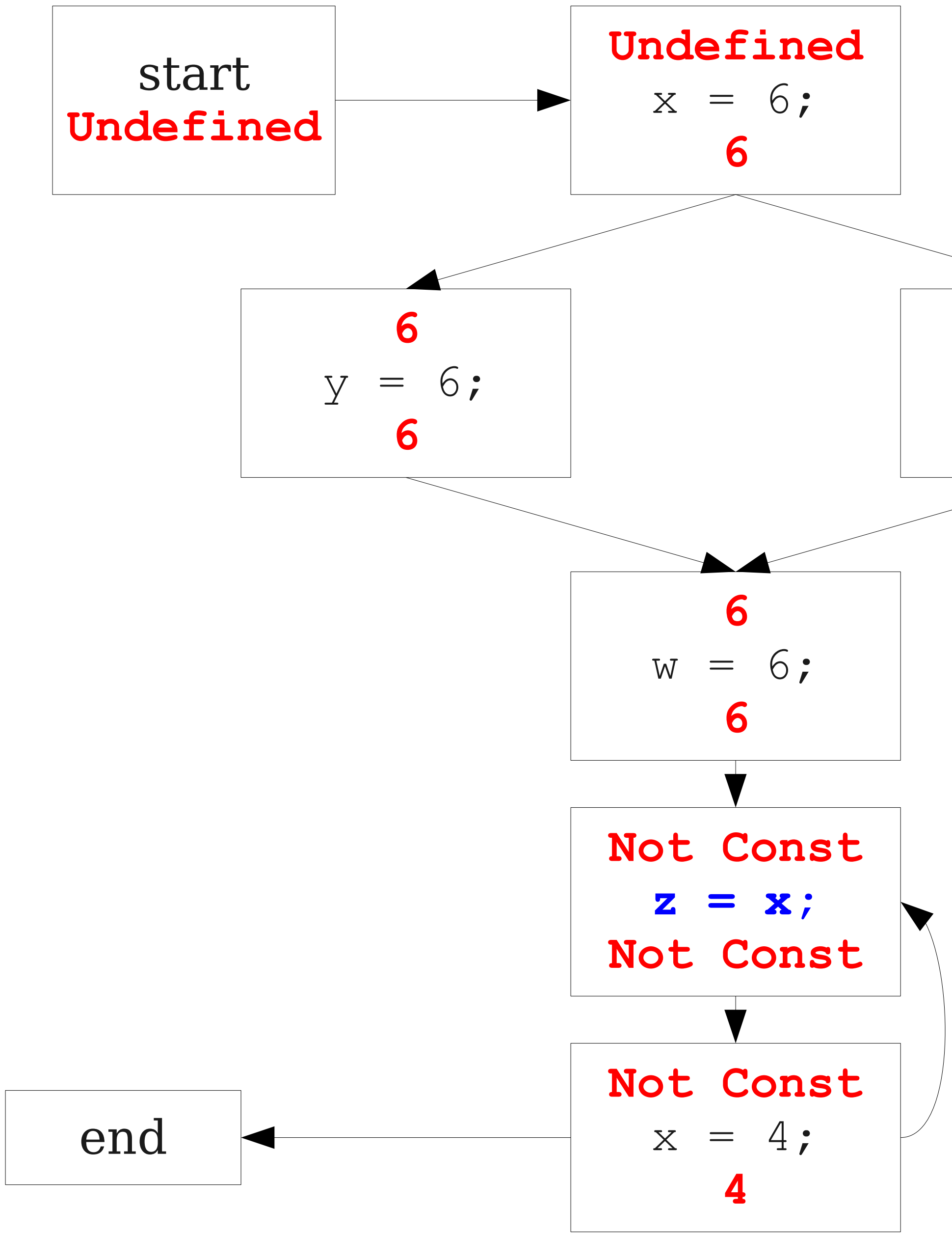

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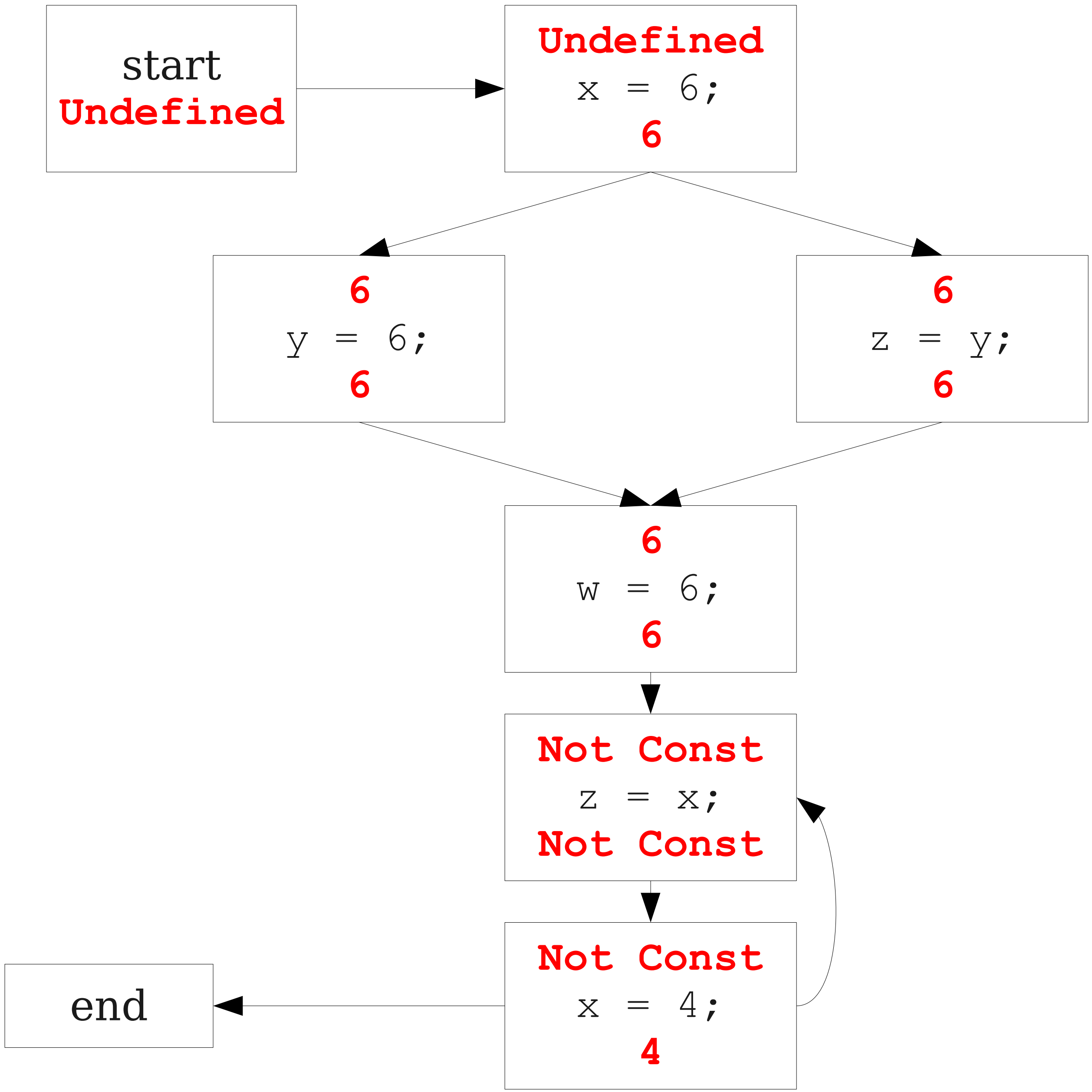

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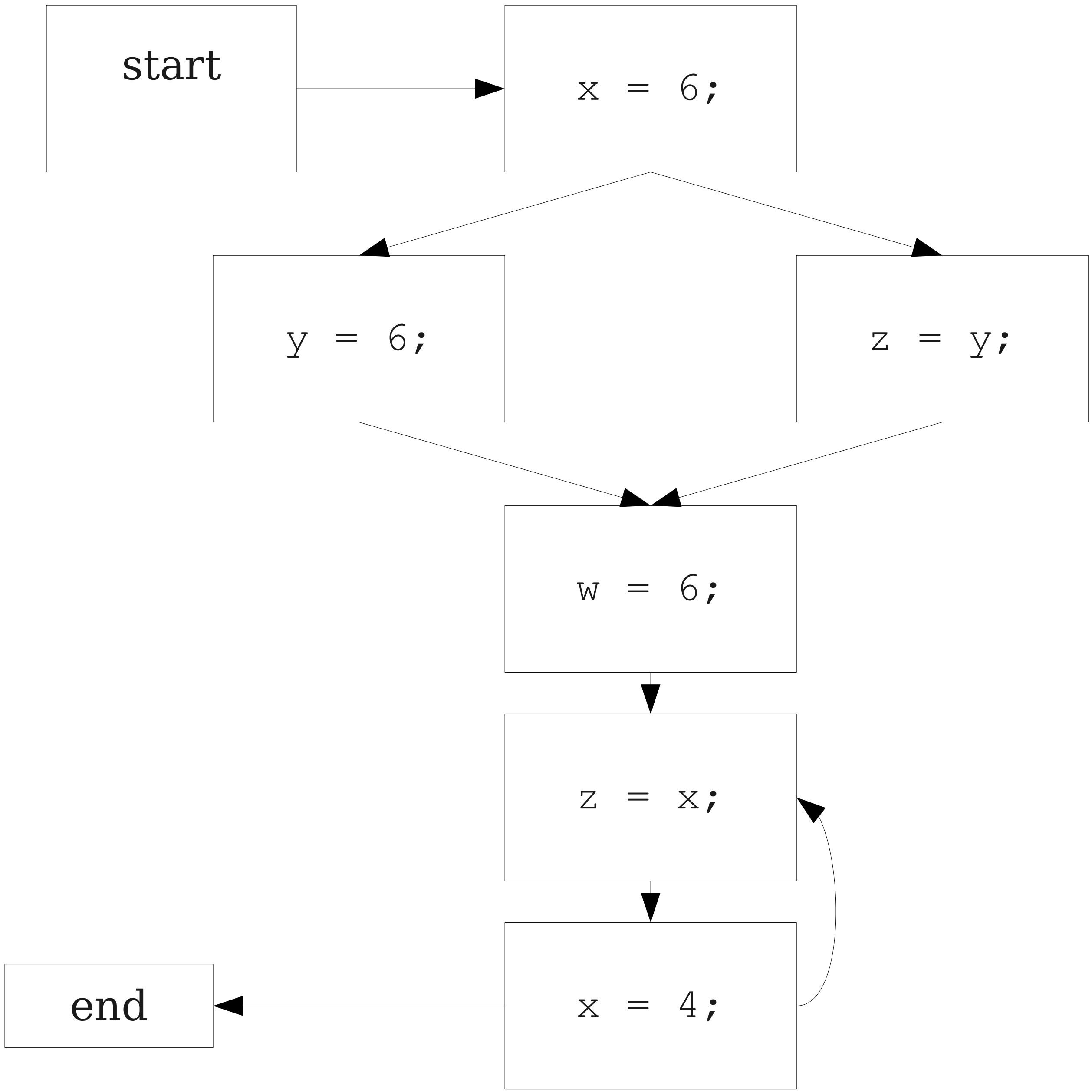

Dataflow for Constant Propagation

- Direction: **Forward**
- Semilattice: **Defined earlier**
- Transfer functions:
  - $f_{x=k}(V) = k$ *(assign a constant)*
  - $f_{x=a+b}(V) =$ Not a Constant *(assign non-constant)*
  - $f_{y=a+b}(V) = V$ *(unrelated assignment)*
- Initial value: **x is Undefined**
  - (When might we use some other value?)